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NATURAL PHILOSOPHY

EXAMINATION PAPERS

R. WORMELL

OUTLINE OF A COURSE

OF

NATURAL PHILOSOPHY,

WITH

SPECIMEN EXAMINATION PAPERS

SUITABLE TO INTERMEDIATE SCHOOLS AND COLLEGES.

BY

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"FELIX QUI POTUIT RERUM COGNOSCERE CAUSAS."—*Virgil.*

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PREFACE.

The following pages contain an outline of the course of lectures I have given, during some years, at the Catholic University of Ireland, together with a selection from the questions and exercises for examination which I have been in the habit of setting to my class. It will be evident to those who are versed in Natural Philosophy that I have dealt, for the most part, with subjects of quite an elementary character, and, as many may think, hardly suited to the dignity of a University Course. I have only to say that my chief desire was to meet the wants of the students who attended my lectures, rather than to set up any high ideal of what a University Course ought to be. The great majority of these students had little or no previous training in Physical Science; and I thought it better for them to begin with the elements, and make some solid progress, than to aim at higher flights, and learn nothing.

Nevertheless, I am strongly of opinion, that such a course of Natural Philosophy as is here sketched out is fairly within the reach of Intermediate Schools and Colleges; and that, at no distant time, it will be expected to form a regular part of their Curriculum. Already the subject finds a place on the Programme of Examinations of the Intermediate Education Board for Ireland. No doubt the Commissioners have contented themselves, for the present, and wisely as I think, with the modest standard of Ganot's Popular Natural Philosophy. But, I trust, it will soon be found possible to adopt a higher standard, such, for example, as Ganot's larger work, at least for what are called the Extra Questions. In the meantime, the few pages I now venture to publish may, perhaps, serve, in some sense, as a guide to the teachers and pupils of Intermediate Schools; assisting them to do what is, at present, demanded of them, and pointing the way to something higher.

In arranging the Examination Papers I have distributed the Course over three years, according to the order set forth in the Programme of the Intermediate Education Board: and in each year there will be found eight monthly and two general examination papers.* The object of these papers is not merely to test the knowledge of the student, but to train him in the art of arranging his knowledge ready for use, and to exercise him in the use of it. It will be seen that each paper is divided into two parts, Questions and Exercises. In the Questions the student is asked to expound some general principle, to describe some experiment, to demonstrate some formula, to discuss some theory, to sketch out some line of reasoning. Accuracy of statement, clearness of exposition, vigour of judgment, quickness of observation, are the qualities here brought into action. In the Exercises, on the other hand, he is called upon to bring his knowledge of general principles to bear on specific problems, with the aid of ordinary arithmetic and the first elements of mathematics; he is taught practically to understand how all the phenomena of Nature, great and small, are capable of accurate measurement and comparison; and he learns with delight that the dry symbols of mathematics, when brought into contact with physical facts, are no longer mere abstractions, but keen and subtle instruments of thought, instinct with life and meaning.

With a view to the interests of those who may be pursuing their studies without the aid of a master, I have thought it well to give some help in the form of brief notes, inserted within brackets, and appended to the Questions and Exercises to which they refer. I would suggest, however, to the young student, that he should make it his practice, before having recourse to these notes, to consider carefully the conditions of the question put, and, if possible, to work out the answer for himself. And I would strongly urge upon him not to be content simply with solving the problem set before him, but to make sure, in

* The Examination Papers for the first two years of the Course appear in the present volume; those of the third year are in preparation, and will be published with as little delay as possible.

each case, that he has mastered the significance of the formulas employed, and is able to demonstrate their truth, if called upon to do so. It is hardly necessary to say that, in the preparation of these examination papers, I have not hesitated to use the materials already existing, in ordinary Text Books, and in published examination papers, whenever I found them suitable to my purpose.

The British system of weights and measures, in common use amongst us, is ill-suited to scientific purposes, and is hardly known on the continent of Europe ; yet it is so interwoven with the habits and language of everyday life, that it is practically impossible to get rid of it. This is a misfortune for the student of science in these islands : for it doubles, in effect, a portion of his work. He must master the metric system of weights and measures, because it is so convenient for calculation, and because it is so generally used by scientific men ; whilst, at the same time, he must have a perfect command of the British system, if he wishes to apply his scientific knowledge to the practical purposes of life. Hence in setting examination papers, I have always thought it desirable to exercise the student in both systems; believing that by constant exercise only could he acquire a thorough knowledge of both, and of their mutual relations.

It will be observed that I have adhered to the old-fashioned names of Mechanics, Hydrostatics, and Pneumatics. I have done so, simply because these names are in possession, as representing certain well-known branches of Natural Philosophy. Every one now admits that they are open to great objection, and I feel satisfied they must, in the end, be discarded: but unfortunately, up to the present time, no one system of nomenclature has been adopted in their stead, with anything approaching unanimity, by scientific men. The examination papers I have set on these subjects require an elementary knowledge of Theoretical Mechanics, which belongs to a separate Division of the Intermediate Education Programme. They will be easily answered by any student, who has read Haughton's or Newth's Mechanics, and Besant's Hydrostatics.

At the end of the volume, I have given, in Appendix I., the Examination Papers, on the subjects of Theoretical Mechanics and Natural Philosophy, issued by the Board of Intermediate Education for Ireland, in June, 1879. These papers fairly correspond, in point of difficulty, with the standard of Ganot's Popular Natural Philosophy; while my papers correspond approximately with the standard of Ganot's larger work. The former standard represents, it may be, all that is expected just now from Intermediate Schools: but the latter, I feel confident, more nearly represents what will, very generally, be demanded of them a few years hence.

In a second Appendix will be found a short account of a new form of Voltaic Battery, which I devised, some eighteen months ago, for my own use, and which, I think, is more convenient for the ordinary purposes of College lectures than any other battery with which I am acquainted. A third Appendix contains a list of books on Natural Philosophy, which may be regarded as trustworthy guides to teachers and students, and which would constitute a good beginning for a collection, in this department of knowledge, suitable to School and College libraries.

In conclusion, I desire to record my obligations to Mr. Kelly, of Terenure College, one of my pupils, who, with great kindness, has checked the answers to the exercises, and who has also assisted me in correcting the proof sheets. But notwithstanding the pains we have taken to secure accuracy, it is hardly to be hoped that a book containing so large a proportion of figures and mathematical symbols should be altogether free from errors; and I shall esteem it a favour, on the part of any of my readers, who will kindly let me know of the mistakes or the misprints he may happen to detect.

G. M.

November 24th, 1879.

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OUTLINE OF A COURSE
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SPECIMEN EXAMINATION PAPERS.

FIRST YEAR'S COURSE:

MECHANICS, HYDROSTATICS, PNEUMATICS.

MONTHLY EXAMINATION PAPER, No. 1.

MECHANICS, Sections I., II., III.

A. Questions.

1. Give some striking examples of the divisibility of matter.

2. Give accurate definitions of the English Yard and of the French Metre, illustrating your definitions by reference to the history of each. Write down the equivalent of a Metre in English Inches, of a Litre in English Pints, and of a Kilogramme in English Pounds Avoirdupois.

3. State briefly the advantages of the metric system of weights and measures; and give examples to show its superiority in point of convenience, as compared with the English system.

4. Explain and illustrate the doctrine of the parallelogram of forces.

5. Show how to find the resultant of three given forces acting on a point; and prove that to produce equilibrium their directions must lie in the same plane. (London University, Matriculation, January, 1870).

6. When a horse draws a canal boat is it better to use a long rope or a short rope; and why? (Ditto, June, 1872).

7. Show how it is possible for a sailing vessel to be driven by the wind in a direction different from that in which the wind is blowing. (Ditto, June, 1872).

8. A heavy plummet is immersed in a stream, the string being held by a person standing on the bank. The string is found to settle in a sloping position. Show, by means of a sketch, the three forces which keep the plummet in equilibrium. (Ditto, June, 1873).

9. Explain fully what is meant by the Centre of Parallel Forces, and apply this explanation to the Centre of Gravity of a material body.

10. Describe an experimental method of finding the Centre of Gravity of a thin plate, and explain the reasoning on which this method is founded.

11. Define the terms Stable, Unstable, and Neutral equilibrium; state the conditions of each, and give examples.

12. Explain fully, with the aid of a diagram, why a cart is more easily upset when loaded with 10 cwt. of hay than when loaded with 10 cwt. of lead.

B. **Exercises.**

1. A tank of distilled water is 5 metres long, 3 metres wide, and 2 metres high. Write down its volume in litres, and its weight in kilogrammes; also the weight, in kilogrammes, of an equal volume of masonwork whose specific gravity is 2·5. *Ans.* (1) 30,000; (2) 30,000; (3) 75,000.

2. Two forces, of 45 lbs. and 35 lbs. respectively, act on a point, at an angle of 90°; find their resultant. *Ans.* 57 lbs. nearly.

3. The resultant of two forces acting on a point, at an angle of 90°, is 85 lbs., and one of the forces is 77 lbs.; find the other force. *Ans.* 36 lbs.

4. The resultant of two equal forces acting on a point, at an angle of 90°, is 100 lbs.; find the magnitude of each force. *Ans.* 70·7107.

5. Two forces, of 9 lbs. and 56 lbs. respectively, act on a point, at an angle of 60°; find the resultant. *Ans.* 61 lbs.

6. Three equal forces acting on a point are in equilibrium; show that the angle between any two of them is 120°.

7. Two forces, of 65 and 84 lbs. respectively, act on a point, at an angle of 70°; find their resultant. *Ans.* 122·54 lbs.

[$R^2 = P^2 + Q^2 + 2PQ \times \cos\theta = (65)^2 + (84)^2 + 2 \times 65 \times 84 \times \cdot 342 = 15015{\cdot}64$. Hence $R = \sqrt{15015{\cdot}64} = 122{\cdot}54$.]

8. A boat is pulled along a canal, 50 feet wide, by men on each bank; the length of each rope from its point of attachment to the bank is 72 feet; the boat moves straight down the middle of the canal; find the total effective pull in that direction, when the pull on each rope is 7 cwt. *Ans.* 13·1292 cwt.

[Make a diagram representing the conditions of the problem. You will have two equal right angled triangles, in each of which the hypotheneuse, which corresponds to a rope, is 72 feet, one of the small sides, which corresponds to half the breadth of the canal, is 25 feet, and the other small side represents the direction in which the boat is moving. Calling the angle each rope makes with this direction θ, you will find the effective pull of each rope to be, in cwts., $7 \times \cos\theta$. Now, from the diagram, $\sin\theta = \frac{25}{72} = {\cdot}3472$; and the angle whose sine is ·3472 you will find in the tables to be 20° 19′. The cosine of 20° 19′ is ·9378, and therefore $7 \times \cos\theta = 7 \times {\cdot}9378 = 6{\cdot}5646$. Hence the effective pull of each rope is 6·5646 cwt., and the total effective pull is $6{\cdot}5646 \times 2 = 13{\cdot}1292$ cwt.

The problem may be solved, without the aid of tables, thus:—If x represent the third or unknown side of each of the right angled triangles, $x^2 = (72)^2 - (25)^2$, from which $x = 67{\cdot}52$; and if the actual pull on each rope be represented by the hypotheneuse, the effective pull (say f) will be represented by x. Therefore $f : 7$ cwt. $:: 67{\cdot}52 : 72$, from which $f = \frac{7 \times 67{\cdot}52}{72} = 6{\cdot}56\dot{4}$, and the total effective pull $2f = 13{\cdot}129$.]

9. A weight of 5 lbs. is suspended freely from a fixed point by a flexible string; find what horizontal force, applied to the weight, will keep the string inclined at an angle of 30° to the vertical line. *Ans.* 2·88 lbs.

[Make a diagram representing the conditions of the problem. You will see that the heavy body is kept at rest by the action of three forces, namely, its own weight W, the tension of the string T, and the horizontal force H; these forces will be represented by the sides of a right angled triangle, and from the diagram you can prove that $T = 2H$. Further $H^2 = T^2 - W^2 = (2H)^2 - 5^2 = 4H^2 - 25$. Hence $3H^2 = 25$ and $H = \frac{5}{\sqrt{3}} = 2{\cdot}88$.]

10. Two weights, of 126 lbs. and 220 lbs. respectively, are suspended from the extremities of a straight bar, 26 inches in length; find the point at which their resultant will intersect the bar. *Ans.* 9·47 inches from one end, 16·53 from the other.

[Call the segments into which the bar is divided by the resultant, a and b, the weights, P and Q; then $a + b = 26$, and $Pa = Qb$. From these equations a and b can be found.]

11. A uniform cube is placed on a table, with its edges parallel to the edges of the table. The table is then tilted up, and the cube is prevented by friction from slipping. At what inclination of the table will it be just on the point of toppling over? *Ans.* 45°.

12. A homogeneous right cylinder, 6 inches in diameter, which rests on a rough inclined plane, is just on the point of toppling over,

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when the angle of the plane's inclination is 60°; find the height of the cylinder. *Ans.* 3·4641 inches.

[Make a diagram representing the conditions of the problem. The centre of gravity of the cylinder is the middle point of its axis. Through this point draw a vertical line, which, since the cylinder is on the point of toppling over, must pass through that point of the base which is lowest on the inclined plane. Now, let fall a perpendicular from the centre of gravity of the cylinder on its base: and it will be easy to prove from the diagram, either by Geometry or Trigonometry, that the height of this perpendicular is 1·73205 inches, and, therefore, that the height of the cylinder is 3·4641 inches.]

13. A bar, 9 feet long, to which is attached a weight of 150 lbs., at a point 3 feet from one end, is borne by two men. Find what portion of the weight is supported by each man, when the bar is horizontal. *Ans.* 100 lbs. by one, 50 lbs. by the other.

14. A homogeneous bar, 5 feet long, weighs 10 lbs.; weights of 9 lbs. and 5 lbs. respectively are suspended from its extremities; find the point on which it will balance. *Ans.* 5 inches from the middle point.

[Make a diagram representing the conditions of the problem. The bar, left to itself, would balance on the middle point, 30 inches from each end; but, when the weights are added, it will evidently balance on a point nearer to that end to which the heavier weight is attached. Let x be the distance of this point from the middle point. We may suppose the mass of the bar to be concentrated at its centre of gravity, which is the middle point, and the whole system connected together by a rigid rod without weight. Then, when the system is balanced, we should have $5(30 + x) + 10x = 9(30 - x)$, or $150 + 5x + 10x = 270 - 9x \therefore 24x = 120$, and $x = \frac{120}{24} = 5$.]

MONTHLY EXAMINATION PAPER, No. 2.

MECHANICS, SECTIONS IV., V.

A. Questions.

1. Explain the three orders of Lever and give examples of each.

2. Define what is meant by the sensibility of a balance; and show how it depends (1) on the length of the beam, (2) on the weight of the beam, and (3) on the distance of the centre of gravity from the fulcrum.

3. State and prove the condition of equilibrium in the Wheel and Axle.

4. State and prove the condition of equilibrium on an Inclined Plane, neglecting friction, when the power acts parallel to the base.

5. Describe the Screw, and show its relation to the Inclined Plane.

6. Explain and demonstrate the mechanical advantage of the Screw.

7. Define the terms Weight and Mass; explain clearly and fully the difference between them, and illustrate your answer by examples.

8. How is the magnitude of a force measured? What is meant by *gravitation* units of force, and what by *kinetic* units of force?

[The magnitude of a force, like every other magnitude, is measured by units; and the unit employed must be some particular force which is itself clearly defined and always the same. Two kinds of unit of force are in use among scientific men: the one may be called the *gravitation* unit, the other the *kinetic* unit. The gravitation unit is the force with which gravity pulls down the unit of mass. In these countries the unit of mass is the pound weight; and, therefore, the British gravitation unit is the force with which gravity pulls down the pound weight. This force is always the same for the same place; and it is very nearly the same for different parts of a small country like England. Hence it is in general use where great accuracy is not required; as, for instance, in measuring the pressure of steam in a boiler. But since the value of this unit varies with the force of gravity, and the force of gravity varies with the latitude, it is not suitable for comparing together forces at different parts of the Earth's surface, when great accuracy is required. The *kinetic* unit of force is the force which, acting on a unit of mass, for a unit of time, would generate a unit of velocity. Hence in England it is the force which, acting for a second, on a pound weight, would give it a velocity of one foot a second. This unit is every where exactly the same, and may be used for comparing together forces acting at different parts of the Earth. In the metric system the *gravitation* unit is the force with which gravity pulls down a kilogramme; and the *kinetic* unit is the force which, acting for a second, on a gramme weight, would give it a velocity of one centimetre a second.]

9. Show how gravitation units of force, at any place, may be converted into kinetic units, and kinetic units into gravitation units.

[This conversion may be made when the value of the force of gravity, at the place in question, has been exactly determined. Thus, for example, the force of gravity, in London, acting for a second, on a pound weight, gives to it a velocity of 32·19 feet a second; that is to say, it pulls down a pound weight with a force which is 32·19 times greater than the kinetic unit of force. In other words, the British gravitation unit, in London, is equal to 32·19 British kinetic units. Hence to convert British gravitation units, in London, into British kinetic units, we must multiply by 32·19; and to convert British kinetic units into British gravitation units we must divide by 32·19. Again, gravity acting for a second on a gramme weight, in Paris, would give it a velocity of 981 centimetres a second; therefore the metric gravitation unit, in Paris, is equal to 981 metric kinetic units; and to convert gravitation units into kinetic units we must multiply by 981, while to convert kinetic units into gravitation units we must divide by 981.]

10. State the First Law of Motion; and give a summary of the evidence, direct and indirect, by which it is established.

11. A ship is moving forwards in a straight line, and a stone is dropped from the top of the mast. Will the stone fall at the foot of the mast or behind it? Indicate the path of the stone by a diagram, and give a reason for your answer.

12. If a person is moving forward, at a uniform rate, in a straight line, in what direction must he throw a ball upwards, if he want it to return to his hand; and why?

13. If a ball is thrown out of the window of a railway carriage, in a direction at right angles to the motion of the train, in what direction will it seem to fall, and in what direction will it really fall?

14. State the Third Law of Motion, and show how it holds in the case of a stone which is in the act of falling towards the earth. (London University, 1873).

B. Exercises.

1. A weight of 56 lbs. is attached to one end of a uniform bar, which is 10 feet long, and weighs 20 lbs.; the fulcrum is 2 feet from the end to which the weight is attached; what power must be applied at the other end to balance the weight? *Ans.* 6·5 lbs.

[Make a diagram representing the conditions of the problem. The bar may be conceived to be concentrated at its centre of gravity, the middle point, and the whole system may be supposed to be connected together by a rigid rod without weight. We shall then have, calling the power P, and the weight W, $8P + 20 \times 3 = 56 \times 2$. Hence $8P = 112 - 60 = 52$, and $P = \frac{52}{8} = 6{\cdot}5$.]

2. Find the true weight of a body which, when placed in one scale of a false balance (arms of unequal length), weighs 140 grammes, and when placed in the other, weighs 154·35 grammes. *Ans.* 147 grammes.

3. A weight of 247 lbs. is attached to one end of a horizontal straight lever, which is 22 inches long, and may be regarded as having no weight; the power is applied at the other end and makes an angle of 27° with the lever; the fulcrum is 3 inches from the weight. Find the magnitude of the power when it just balances the weight. The sine of 27° is ·454. *Ans.* 85·9 lbs.

[Make a diagram representing the conditions of the problem. Let P and W represent the power and weight, a and b the arms of the lever, α and β the angles which the power and weight make respectively with the arms of the lever, then $P \times a \times \sin \alpha = W \times b \times \sin \beta$, or $P \times 19 \times \cdot454 = W \times 3 \times 1$. Hence $P = \frac{3W}{19 \times \cdot454} = \frac{741}{8\cdot626} = 85\cdot9$.]

4. The circumferences of a Wheel and Axle are respectively 36 and 15 inches; what power applied at the circumference of the wheel will support a weight of 30 cwt. suspended to the circumference of the axle? *Ans.* 12·5 cwt.

$$[P : W :: r : R :: c : C :: 15 : 36 \therefore P = \frac{15W}{36} = \frac{450}{36} = 12\cdot5.]$$

5. Sixteen sailors, exerting each a pressure of 29 lbs., turn a capstan with a length of lever equal to 8 feet, the radius of the capstan being one foot two inches. Find the weight that this force is capable of supporting. *Ans.* 1 ton, 8 cwt., 45 lbs.

6. A weight is just supported on a smooth inclined plane of 1 in 5, by a force of 10 lbs., acting parallel to the plane; what is the magnitude of the weight? What would it be if the power acted parallel to the base? *Ans.* (1) 50 lbs., (2) 49 lbs. nearly.

$$[(1)\ P : W :: H : L :: 1 : 5 \therefore W = P \times 5 = 50.$$
$$(2)\ P : W :: H : B :: 1 : 4\cdot9 \therefore W = P \times 4\cdot9 = 49.]$$

7. A railway train weighs 100 tons, and a force of 1 ton is necessary to move it on a level; what force will be required to move it up an incline of 1 in 27? *Ans.* 4·7037 tons.

[If there were no friction, then, for equilibrium on the inclined plane, $P = W\frac{H}{L} = 100 \times \frac{1}{27} = 3\cdot7037$. Now if to this force be added one ton, which would just overcome friction and produce motion on the level, the train will move up the incline. Therefore the force required is $3\cdot7037 + 1 = 4\cdot7037$.]

8. The angle of inclination of a smooth inclined plane is 45°; a force of 3 lbs. acts horizontally, and a force of 4 lbs. acts parallel to the plane. Find the weight which both together will be just able to support. *Ans.* 8·6568 lbs.

[Make a diagram and show that the height (H) of the plane is equal to its base (B), and that the length (L) is equal to either multiplied by $\sqrt{2}$. The horizontal force (P) will support a weight (W) given by the formula $W = P\frac{B}{H} = P = 3$ lbs. The other force (P′) will support a weight (W′) given by the formula $W' = P'\frac{L}{H} = 4 \times \frac{H \times \sqrt{2}}{H} = 4 \times \sqrt{2} = 5\cdot6568$. Therefore both together will support a weight equal to $W + W' = 3 + 5\cdot6568 = 8\cdot6568$.]

9. A rope, one end of which is attached to a fixed beam, passes under a moveable pulley, then over a fixed pulley, and hangs down loose. To the moveable pulley is attached a board, on which a man sits, holding the loose end of the rope. Neglecting friction and the weight of the rope, find the force with which the man must pull vertically downwards in order to support himself, the man, board, and moveable pulley weighing together 180 lbs. (London University, 1869). *Ans.* 60 lbs.

10. Suppose, in the above exercise, that the rope after passing over the fixed pulley in the beam above, passes under another fixed pulley below; and that the loose end of the rope then comes up to the man sitting on the board; find the force with which he must pull vertically upwards in order to support himself. (Ditto, 1869). *Ans.* 180 lbs.

[The student will observe, in the two preceding exercises, an interesting illustration of the Third Law of Motion. When the man pulls the rope downwards, the tension of the rope pulls him upwards, and relieves the board of a portion of his weight equal to the tension of the string, that is, equal to his downward pull. Again, when the man pulls the rope upwards, the tension of the rope pulls him downwards with exactly equal force, and therefore increases the pressure on the board by that amount. Let the student make a diagram representing the conditions of the problem, and, calling the force of the pull, in each case, x, he will get, in the first case, $x = \frac{180 - x}{2}$, or $2x = 180 - x, \therefore 3x = 180$, and $x = 60$; in the second case $x = \frac{180 + x}{2}$, or $2x = 180 + x, \therefore x = 180$.]

11. In the first system of pulleys, with 3 moveable pulleys, each weighing 80 grammes, what power will balance a weight of 600 grammes, (1) if friction and the weight of the pulleys be neglected, (2) if friction be neglected, but not the weight of the pulleys? *Ans.* (1) 75 grammes; (2) 145 grammes.

[If P represent the power, W the weight, w the weight of each pulley, and n the number of moveable pulleys, then, in the first case $P = \frac{W}{2^n} = \frac{600}{8} = 75$; in the second case, $P = \frac{W + (2^n - 1)\,w}{2^n} = \frac{600 + 560}{8} = \frac{1160}{8} = 145$.]

12. If the circumference of a screw is 10 inches, and the distance between the threads $\frac{1}{4}$ inch, what power must be applied to overcome a resistance of 30 lbs.? *Ans.* 12 ounces.

13. A ship is sailing due north at the rate of 4 knots an hour, and a ball is projected due east along her deck, at the rate of 10 feet

a second. Find the actual motion of the ball, compounded of these two motions. *Ans.* Direction 34° 1′ north of east, velocity 12·065 a second.

[A knot = 2025 yards. The ship goes this distance in 15 minutes, or $\frac{2025}{15} =$ 135 yards = 405 feet a minute, which is $\frac{405}{60} = 6{\cdot}75$ feet a second. Now make a diagram representing the two velocities at right angles to one another and complete the parallelogram. We have thus $R^2 = (10)^2 + (6{\cdot}75)^2 = 145{\cdot}5625$, and $R = 12{\cdot}065$. To find the direction of the resultant: let a be the velocity eastward, b the velocity northward, and θ the angle which the resultant makes with the former; then $\tan\theta = \frac{b}{a} = \frac{6{\cdot}75}{10} = {\cdot}675$, and in the tables we get the angle whose tangent is ·675, 34° 1′.]

MONTHLY EXAMINATION PAPER, No. 3.

MECHANICS, SECTIONS VI., VII.

A. Questions.

1. State the law of universal gravitation; give a brief sketch of the history of its discovery, and point out the evidence by which it has been demonstrated.

2. Give an account of the means by which Galileo first showed that the force of gravity acts on all bodies alike; and then describe the more exact experiments devised by Newton to demonstrate the same truth.

3. Describe Attwood's machine, and state the laws of falling bodies that may be established by its aid.

4. Prove that if a body be projected vertically upwards, with any velocity, the height to which it will rise is equal to the distance through which it should fall to acquire the same velocity.

5. If f represents the force that impels a body down an inclined plane, g the force of gravity, α the angle the inclined plane makes with the horizon, R the pressure on the plane, H, L, and B the height, length, and base of the plane respectively, prove the formulas, $f = g\ \sin\alpha = g\,\frac{H}{L}$, $R = g\ \cos\alpha = g\,\frac{B}{L}$.

6. Describe the experiments by which Galileo investigated the laws of falling bodies, by means of the inclined plane.

7. Explain what is meant by centrifugal force, and show that it is not a force properly so called. Give some examples of centrifugal force in the experience of every day life.

8. Explain and illustrate with a diagram the influence of centrifugal force on the weight of bodies, and on the shape of the earth.

9. Explain why it is that a horse going round in a circus leans inwards towards the centre, and that the faster he goes the more he inclines.

10. It is usual on a sharp curve, on railways, to raise one rail above the level of the other; explain, as fully as you can, the mechanical reason for so doing.

11. Show how it may be proved that the value of centrifugal force at the Equator is ·11126; and deduce from this that the total force of gravity, at the Equator, is 289·4 times as great as centrifugal force, the apparent force of gravity being 32·088.

12. How do you account for this apparent contradiction: the unit of weight is a pound, and the unit of mass is a pound; therefore a body has the same number of units of weight and of mass, in other words $W=m$; and yet we constantly use the formula $W=mg$, which implies that the weight is 32 times as great as the mass?

[The word *weight* has three meanings which it is important to distinguish from one another. In common use it means the *number of pounds* a body weighs, *as representing the quantity of matter* it contains. This meaning is hardly to be distinguished from the meaning of the word *mass* among scientific men; and in this sense evidently $W = m$. Next, the word *weight* is used to signify the *number of units of force* with which gravity pulls a body downwards. This is the true scientific sense of the word. But it may be understood in two ways. If the *gravitation unit* of force be adopted, then $W = m$; for the pound weight is at the same time the gravitation unit of force and the unit of mass, and therefore the number of gravitation units of force with which gravity pulls a body down is always the same as the number of units of mass that the body contains. But if the *kinetic unit* of force be adopted, then $W = mg$; for gravity pulls down each unit of mass with a force g, equal to about 32 kinetic units, and therefore it pulls down a mass of m units with a force mg equal to about $32m$.]

B. Exercises.

[Formulas for the solution of problems on the laws of falling bodies. If S represent the space passed over in any time t, V the velocity at the end of that time, and g the force of gravity, then, when the body falls from rest,

$$(1)\ V = gt;\ (2)\ S = \tfrac{1}{2} gt^2;\ (3)\ V^2 = 2g\,S.$$

If the body, when gravity begins to act on it, has already a velocity downwards V′, then

$$(4)\ V = V' + gt;\ (5)\ S = V't + \tfrac{1}{2} gt^2;\ (6)\ V^2 = V'^2 + 2g\,S.$$

Finally, if the body, when gravity begins to act on it, has a velocity upwards V', then

$$(7)\ V = V' - gt;\quad (8)\ S = V't - \tfrac{1}{2}gt^2;\quad (9)\ V^2 = V'^2 - 2g\,S.$$

The student should satisfy himself that he fully understands and is able to demonstrate each of these formulas. In the exercises that follow g may be taken equal to 32, and friction may be neglected.]

1. Through what space must a body fall to acquire a velocity of 80 feet a second? *Ans.* 100 feet.

$$[V^2 = 2g\,S \therefore S = \frac{V^2}{2g} = \frac{6400}{64} = 100.]$$

2. A body falls freely from rest for 6 seconds. What space will be described in the last second? *Ans.* 176 feet.

[Space described in 6 seconds $S = \frac{1}{2}gt^2 = 16 \times 36 = 576$. Space described in the first 5 seconds $S = \frac{1}{2}gt^2 = 16 \times 25 = 400$. Space described in the sixth second $576 - 400 = 176$.]

3. A stone is dropped from the top of a cliff, 784 feet high; how long will it take to fall to the bottom of the cliff, and what will be its velocity when it arrives there? *Ans.* (1) 7 seconds; (2) 224 feet a second.

4. If, in the last exercise, the stone were projected downwards with a velocity V', and were found to reach the bottom of the cliff in 5 seconds, what would be the value of V'? *Ans.* 76·8 feet a second.

[$S = V't + \frac{1}{2}gt^2 = 5\,V' + 16 \times 25$; hence $784 = 5\,V' + 400 \therefore 5\,V' = 384$, and $V' = \frac{384}{5} = 76{\cdot}8$.]

5. How long must a body fall, under the action of gravity, to acquire a velocity of 128 feet a second? *Ans.* 4 seconds.

6. A lead bullet is dropped down the shaft of a mine, and, after a second, another is dropped from the same point; what will be the distance between them when the first has been falling for 5 seconds? *Ans.* 144 feet.

7. A stone is projected vertically upwards, with a velocity of 100 feet a second; find the time when it will be at a height of 156 feet. *Ans.* In 3 seconds, and again in $3\frac{1}{4}$ seconds.

[$S = V't - \frac{1}{2}gt^2$, that is, $156 = 100t - 16t^2$, or $16t^2, - 100t = -156$; and solving this equation we get $t = 3$ or $3\frac{1}{4}$. The student will observe the significance of the two roots of the quadratic; the stone will pass the same point *twice*, first on its way up, and again on its way down.]

8. A body projected vertically upwards against gravity has risen 120 feet in one second. What was its initial velocity of projection,

and how far will it rise during the next second? (London University, Matriculation, Jan. 1876.) *Ans.* (1) 136; (2) 88.

[$S = V't - \frac{1}{2}gt^2 = V' - 16$, $\therefore V' = 120 + 16 = 136$. Again, in two seconds, $S = 2V' - \frac{1}{2}g \times 4 = 272 - 64 = 208$; and therefore the space described in the second second will be $208 - 120 = 88$.]

9. A man descending in a mine, at the rate of 100 feet a minute, drops a stone which reaches the bottom in 2 seconds; through what distance did it fall? *Ans.* $67\frac{1}{3}$ feet.

[Here the stone when dropped had already a velocity downwards of 100 feet a minute, or $\frac{100}{60} = 1\frac{2}{3}$ feet a second; hence the formula $S = V't + \frac{1}{2}gt^2$ gives us $S = 1\frac{2}{3} \times 2 + 16 \times 4 = 67\frac{1}{3}$.]

10. A balloon has reached a height of 2000 feet, and continues to rise uniformly with a velocity of 4 feet a second. If a stone be now dropped from the balloon, how long will it take to reach the earth? *Ans.* 11·305 seconds.

[When gravity begins to act the stone has an upward velocity, V', of 4 feet a second. Hence if we regard the downward direction as positive, and the upward direction as negative, we shall have $S = \frac{1}{2}gt^2 - V't$, that is, $2000 = 16t^2 - 4t$; and solving this equation we find $t = 11\cdot305$.]

11. A body is projected up a smooth inclined plane, whose angle of inclination is 30°, with a velocity of 25 feet a second; how far will it travel before it comes to rest? *Ans.* 19·53 feet.

[$V^2 = V'^2 - 2fS$. On the inclined plane $f = g \sin a$, that is, in the present case, $g \sin 30°$, or $g \times \cdot5$; and $V = o$ when the body comes to rest. Hence we have

$$0 = (25)^2 - 2 \times g \times \cdot5 \times S, \text{ or } gS = 625, \text{ and } S = \frac{625}{32} = 19\cdot53.]$$

12. How long will a body take to fall down a smooth inclined plane, 20 feet in length, which makes an angle of 30° with the horizon? *Ans.* 1·581 seconds.

[$S = \frac{1}{2}ft^2$, that is, $20 = \frac{1}{2} \times g \sin 30° \times t^2 = \frac{g}{4}t^2$; hence $80 = gt^2$, and $t^2 = \frac{80}{32} = 2\cdot5$ $\therefore t = \sqrt{2\cdot5} = 1\cdot581$.]

13. A railway carriage weighing 7·21 tons, and moving at the rate of 30 miles an hour, describes a portion of a circle, whose radius is 460 yards; find, in gravitation units, the centrifugal force that acts on the carriage. *Ans.* ·316 of a ton, or 707·84 lbs.

[Formulas for centrifugal force. If v represent the velocity of the body, and r the radius of the circle, then f, the value of centrifugal force acting on one unit of mass, is equal to $\frac{v^2}{r}$; and F, the value of centrifugal force acting on m units of mass, is equal to $m\frac{v^2}{r} = \frac{W}{g} \times \frac{v^2}{r}$. Also, if T be the period of a complete revolution, $f = \frac{4\pi^2 r}{T^2}$, and $F = m\frac{4\pi^2 r}{T^2} = \frac{W}{g} \times \frac{4\pi^2 r}{T^2}$. Here again I would strongly urge on the student the importance of thoroughly mastering the meaning of these formulas, and the proofs by which they are established. In the exercise given above, find $v = 44$ feet a second, and $r = 1380$ feet, then $F = \frac{W}{g} \times \frac{v^2}{r} = \frac{7\cdot21}{32} \times \frac{(44)}{1380} = \cdot316$ of a ton, or $\cdot316 \times 2240 = 707\cdot84$ lbs.]

14. A body weighing 1000 lbs. is constrained to move in a circle whose radius is 100 feet, by means of a string, of which the breaking strain is 450 lbs.; find the velocity of the body at the moment the string breaks. *Ans.* 38 feet a second.

[The magnitude of the centrifugal force acting on the body, at the moment the string breaks, must be, in gravitation units, 450 lbs.; and since $F = \frac{W}{g} \times \frac{v^2}{r}$ we get, at that moment $450 = \frac{1000}{32} \times \frac{v^2}{100} = \frac{10v^2}{32}$; hence $v^2 = 45 \times 32 = 1440$, and $v = 38$ nearly.]

15. Calculate in kinetic units the centrifugal force of a unit mass, moving with a velocity of 361 feet a second, in a circle the radius of which is 17 yards. *Ans.* 2555.3.

$$[f = \frac{v^2}{r} = \frac{(361)^2}{51} = \frac{130321}{51} = 2555\cdot3.]$$

16. A unit mass moves in a circle of which the radius is 100 feet; its centrifugal force is 146 kinetic units; find the periodic time. *Ans.* 5·2 seconds.

[$f = \frac{4\pi^2 r}{T^2}$, that is, $146 = \frac{4 \times 9\cdot87 \times 100}{T^2}$; hence $T^2 = \frac{3948}{146}$, and $T = \sqrt{27\cdot04} = 5\cdot2$.]

17. Calculate the effect of centrifugal force in diminishing gravity for the latitude of 60°, and also the component of centrifugal force that tends to transport matter towards the equator, at the same latitude. *Ans.* (1) ·027805; (2) ·04817558.

[The component diminishing gravity $f \cos^2 l$, and the transporting component is $f \cos l \sin l$, f being the value of centrifugal force at the Equator, and l the latitude. Hence for latitude 60°, the first is $\cdot11126 \times (\cdot5)^2 = \cdot027805$, and the second is $\cdot11126 \times \cdot5 \times \cdot866 = \cdot04817558$.]

MONTHLY EXAMINATION PAPER, No. 4.

MECHANICS, SECTIONS VIII., IX.

A. Questions.

1. State the laws of the simple pendulum, and explain how they may be illustrated by experiment.

2. What is meant by a compound pendulum? Explain how the laws of the simple pendulum may be applied to the compound pendulum.

3. Explain, as fully as you can, how the value of the force of gravity, at any place, may be determined by means of the pendulum.

4. Describe and explain the celebrated pendulum experiment, made in the church of the Panthéon, Paris, by which Foucault demonstrated the rotation of the earth on its axis.

5. Define what is meant by Energy, and explain the difference between Kinetic and Potential Energy, illustrating your answer by examples.

6. If m represent the mass of a body, v its velocity, W its weight, and g the force of gravity, prove that its kinetic energy is $\frac{mv^2}{2}$ or $\frac{Wv^2}{2g}$.

7. Explain the doctrine of the Conservation of Energy, and give a summary of the reasoning by which it is supported.

8. Explain what is meant by mechanical advantage in a machine, illustrating your explanation by examples; and show that, notwithstanding this mechanical advantage, there is always an equality between the work done and the energy expended.

9. Give some general idea of the vast quantity of energy that is poured out every day by the Sun into space.

10. In what sense precisely is it true that a Perpetual Motion machine is impossible, and what are the grounds of this opinion?

B. Exercises.

1. Given the force of gravity in Paris as 32·182, find the length of a pendulum that will vibrate seconds in Paris. *Ans.* 39·127 inches.

[Formulas for the pendulum. If t represent the time of an oscillation, l the length of the pendulum, and g the force of gravity, then

$$(1)\ t = \pi\sqrt{\frac{l}{g}};\ (2)\ l = \frac{gt^2}{\pi^2}.$$

And if L be the length of a pendulum that oscillates once in a second

$$(3)\ L = \frac{g}{\pi^2};\ (4)\ g = L\pi^2.$$

In the exercise given $L = \frac{g}{\pi^2} = \frac{32{\cdot}182}{9{\cdot}87}$ feet $= \frac{386{\cdot}184}{9{\cdot}87} = 39{\cdot}127$ inches.]

2. From the experiments of Kater and Sabine, the length of a pendulum that vibrates seconds in London has been shown to be 39·139 inches; calculate from this the value of the force of gravity in London. *Ans.* 32·1905.

3. Find, from the above data, the length of a pendulum that will oscillate once in every half second in London. *Ans.* 9·78475 inches·

4. In Foucault's famous pendulum experiment the time of an oscillation is said to have been 8 seconds; find the length of the pendulum, the value of g in Paris being 32·18. *Ans.* 208·66 feet.

5. A train weighing 10 tons moves, on a level railway, at the uniform rate of 30 miles an hour; if the resistance of friction and the air is equal to 10 lbs. a ton, find how much work will be done by the engine in 20 minutes. *Ans.* 5,280,000 foot lbs.

[The velocity of 30 miles an hour once acquired will remain constant unless diminished by resistance or increased by the driving power. In the case before us it remains constant; and therefore the total work done by the engine is simply to overcome the resistance. This is 10 lbs. per ton, or 100 lbs., and, in 20 minutes, it is overcome through a distance of 10 miles. Hence the work done is $100 \times 1760 \times 3 \times 10 = 5,280,000$ foot pounds.]

6. What horse power is required to move a train, weighing 48 tons, up an incline of 1 in 120, at the rate of 21 miles an hour, when the resistance of friction and the air is 10 lbs. a ton?

[Suppose the train to be already moving at the rate of 21 miles an hour, which is 1848 feet a minute. The work to be done every minute, in order to keep it going at that rate is (1) to overcome a resistance of 10 lbs. a ton, or 480 lbs., acting through a distance of 1848 feet, that is, $480 \times 1848 = 887,040$ foot lbs.; and (2) to lift the train weighing 48 tons, or $48 \times 2240 = 107,520$ lbs., through a height of $\frac{1848}{120} = 15{\cdot}4$ feet, that is, $107,520 \times 15{\cdot}4 = 1,655,808$ foot lbs. of work. Hence the total work to be done per minute is $887,040 + 1,655,808 = 2,542,848$ foot lbs., and the horse power required is $\frac{2,542,848}{33000} = 77{\cdot}056$. If the train has been moving at a slower rate than 21 miles a minute, this horse power will be more than sufficient to keep it going at that slower rate, and the excess of power will go to increase the velocity of the train until a speed of 21 miles an hour is attained.]

7. If a man on a treadmill, of which the steps are 9 inches high, weighs 12 stone, and takes one step a second, how much work will he do in an hour? *Ans.* 453,600 foot lbs.

[The number of units of work done will be the weight (in lbs.) multiplied by the vertical height (in feet) through which it is lifted; that is $12 \times 14 \times \frac{3}{4} \times 60 \times 60 = 453{,}600$.]

8. How many units of work are required to raise 4 cwt. of coal from a pit 70 fathoms deep? *Ans.* 180,160 foot lbs.

9. What amount of horse power would be required to raise 2000 cubic feet of water, per hour, from a mine 180 fathoms deep? A cubic foot of water may be taken to weigh 62·5 lbs. *Ans.* 68·18.

10. How many cubic feet of water will an engine, working at 20 horse power, raise in an hour from a mine 80 fathoms deep? *Ans.* 1320.

[Work done by the engine per hour is $33000 \times 20 \times 60$ foot lbs. Work expended in raising one cubic foot of water 80 fathoms is $62{\cdot}5 \times 80 \times 6$ foot lbs. Therefore the number of cubic feet raised per hour will be $\frac{33000 \times 20 \times 60}{62{\cdot}5 \times 80 \times 6} = 1320$.]

11. What must be the horse power of an engine, working 10 hours a day, to raise 240,000 gallons of water per day to a height of 70 feet? A gallon of water weighs 10 lbs. *Ans.* 8·48.

12. A steam hammer, weighing 2 cwts., makes 60 lifts per minute, each lift being 2·5 feet high; find the horse power required to give motion to the hammer. *Ans.* 1·02.

13. At what rate, in miles per hour, will a train of 80 tons be drawn on a level by an engine, working at 70 horse power, the resistance of friction and the air being 8 lbs. a ton? *Ans.* 41·01 miles an hour.

[The velocity will go on increasing as long as the work done by the engine exceeds the work done by the resistance; and when these two become equal the train will continue to move on uniformly with its acquired velocity. Now the work done by the engine, per minute, is 70×33000, and the work done by the resistance is $80 \times 8 \times n$, n being the number of feet passed over in a minute. Hence the maximum velocity will be attained when $70 \times 33000 = 80 \times 8 \times n$, that is when $n = \frac{70 \times 33000}{80 \times 8} = 3609{\cdot}375$ feet $= {\cdot}6835$ of a mile. This rate per minute is equal to ${\cdot}6835 \times 60 = 41{\cdot}01$ miles an hour.]

14. At what rate will an engine of 45 horse power draw a train, weighing 60 tons, up an incline of 1 in 224, the resistance of the air and of friction being 11 lbs. a ton? *Ans.* 13·4 miles an hour.

[In this case the power has not only to overcome the resistance, but it has also to lift the train through a height of $\frac{n}{224}$ feet a minute, n being the distance, in feet, travelled in the minute. The work required per minute to overcome the resistance is, in foot lbs., $60 \times 11 \times n$; and the work required to lift the train through $\frac{n}{224}$ feet a minute is $60 \times 2240 \times \frac{n}{224}$. When the sum of these two is equal to the total work done, per minute, by the engine, the train will have attained its maximum velocity; that is, when $660n + 600n = 45 \times 33000$, or when $n = \frac{45 \times 33000}{1260} = 1178{\cdot}57$; which will be $1178{\cdot}57 \times 60 = 70714{\cdot}2$ feet per hour, or 13·4 miles.]

15. Find the horse power of a locomotive which moves a train, weighing 50 tons, at the rate of 30 miles an hour, on a level road, when the resistance of friction and the air is $\frac{1}{70}$ of the weight. *Ans.* 128.

[If x represent the horse power required $x \times 33000 = \frac{50 \times 2240}{70} \times 2640$, or $33x = 4224$, and $x = 128$.]

16. At Woolwich, in September, 1879, an 80-ton gun discharged a shot, weighing 1760 lbs., with a velocity of 1657 feet a second; find the kinetic energy of the shot. *Ans.* 33,707·7 foot tons.

[The kinetic energy of a moving body is $\frac{Wv^2}{2g}$, which, in this case, is $\frac{1760}{64} \times (1657)^2 = 27{\cdot}5 \times 2{,}745{,}649 = 75{,}505{,}347{\cdot}5$ foot lbs., or 33,707·7 foot tons.]

MONTHLY EXAMINATION PAPER, No. 5.

HYDROSTATICS, SECTION I.

A. Questions.

1. Explain the characteristics of a liquid as distinguished from a solid, and from a gas.

2. Describe an experiment by which it may be proved that, in a liquid, pressure is transmitted equally in all directions.

3. Explain the principle of the Hydrostatic Press, illustrating your explanation with a diagram.

4. Describe and explain the experiment of the Hydrostatic Bellows.

5. Prove that the free surface of a liquid at rest is, at every point, perpendicular to the vertical line passing through that point.

6. How would you find the total pressure of a liquid on the horizontal base of the vessel in which it is contained? Show that it is independent of the shape of the vessel, and of the quantity of the liquid.

7. Prove the general formula, $P = wah$, P being the total pressure on any surface immersed in a liquid, a the area of the surface pressed, h the depth of the centre of gravity of the surface pressed below the free surface of the liquid, and w the weight of the unit volume of the liquid.

8. Explain the principle of the Hydrostatic Paradox, and give some examples of it.

9. Explain the law that a liquid always seeks its own level, giving some practical illustrations of it.

B. Exercises.

1. A piston, having an area of 4 square inches, is inserted into the side of a cubical vessel filled with water; find the pressure produced by the piston on the surface of the vessel, when it is pressed in with a force of 2 lbs., the edge of the cubical vessel being 15 inches. *Ans.* 6 cwt. 1 lb.

[Formula for the transmission of pressure in liquids. If s represent the area of the piston, p the pressure applied, S the area of the surface pressed, and P the pressure upon it, then $P = p\frac{S}{s}$. In the case before us, $S = (15)^2 \times 6 - 4 = 1346$, $s = 4$, $p = 2$ lbs., $\therefore P = 2 \times \frac{1346}{4} = 673$ lbs. = 6 cwt. 1 lb.]

2. Two circular pistons are inserted in the sides of a closed vessel, filled with water; the diameter of the one is 3 inches, of the other 32; a pressure of 9 ounces is applied to the smaller; find the pressure transmitted to the larger. *Ans.* 64 lbs.

[The surfaces of the circular pistons are proportional to the squares of their diameters; that is, if the diameters be represented by D and d respectively, $\frac{S}{s} = \frac{D^2}{d^2}$. Hence the formula $P = p\,\frac{S}{s}$ becomes $P = p\,\frac{D^2}{d^2}$, and, in the case before us, $P = 9 \times \frac{1024}{9} = 1024$ oz. = 64 lbs.]

3. If the diameter of the smaller piston be 3 centimetres, and of the larger 18, what pressure on the smaller will produce a pressure of 900 kilogrammes on the larger? *Ans.* 25 kilogrammes.

$$[P = p\frac{D^2}{d^2} \therefore p = P\frac{d^2}{D^2} = 900 \times \frac{9}{324} = 25.]$$

4. If the area of the smaller piston be 12 square centimetres, and of the larger 360, what will be the pressure on the larger when 5 kilogrammes are applied to the smaller? *Ans.* 150 kilogrammes.

5. The circumferences of the pistons are as 4 to 55, and the pressure on the smaller is 400 grammes; find the pressure on the larger. *Ans.* 75·625 kilogrammes.

$$[P = p\frac{D^2}{d^2} = p\frac{C^2}{c^2} = 400 \times \frac{(55)^2}{4^2} = 400 \times \frac{3025}{16} = 75625 \text{ grammes.}]$$

6. The diameter of the head of a cylindrical cask is 20 inches, and the height of the cask is 30 inches; what is the total pressure on the surface when a piston, one inch in diameter, is pressed in with a force of one pound? *Ans.* 3,199 lbs.

[Total surface of cask $= 2\pi r^2 + 2\pi r \times h$, r being the radius of the bars, and h the height of the cask. This gives for the total surface $2\pi r\ (r + h) =$ 2513·272 square inches; and this diminished by the area of the piston will be the surface pressed. Find the area of the piston $s =$ ·7854 of an inch. Then S $=$ 2513·272 − ·7854 $=$ 2512·4866. Hence $P = p\frac{S}{s} = 1 \times \frac{2512{\cdot}4866}{{\cdot}7854} = 3{,}199.$]

7. Find the liquid pressure on a square centimetre, at a depth of 20 metres in a lake. *Ans.* 2 kilogrammes.

[It is equal to the weight of a column of water the base of which is one square centimetre and the height 20 metres. Such a column will evidently contain 2000 cubic centimetres of water, and will therefore weigh 2000 grammes, or 2 kilogrammes.]

8. A thin triangular plate, whose base is 50 centimetres, and height 30, is immersed in a vessel of water, in a vertical position, having its base level with the surface; find the pressure on the plate. *Ans.* 7·5 kilogrammes.

[The area of the plate is $\frac{50 \times 30}{2} =$ 750 square centimetres, and the depth of its centre of gravity is 10 centimetres; the pressure is, therefore, equal to the weight of a column of water 750 × 10 = 7500 cubic centimetres in volume, or 7500 grammes in weight.]

9. Find the liquid pressure on a rectangular plane, 10 inches by 4, immersed in water, and having its centre of gravity at a depth of 20 inches. *Ans.* 28·935 lbs.

10. Suppose a cubical vessel, the length of whose edge is b, to be filled with any liquid, what is the pressure on one of its sides as compared with the pressure on its base? *Ans.* As 1 to 2.

[If we take w to represent the weight of the unit volume of the liquid, the pressure on the side will be $b^2 \times \frac{b}{2} \times w = \frac{1}{2} b^3 w$; the pressure on the base will be $b^2 \times b \times w = b^3 w$. Hence the ratio of the pressures will be $\frac{1}{2} b^3 w : b^3 w$, or 1 : 2.]

MONTHLY EXAMINATION PAPER, No. 6.

HYDROSTATICS, Section II.

A. Questions.

1. Explain the principle of Archimedes, and show how it may be proved both theoretically and practically.

2. What is meant by the centre of buoyancy of a floating body? State and prove the conditions of stable equilibrium for a body totally immersed in a liquid.

3. Describe and explain fully the process of finding the specific gravity of a body by means of the Hydrostatic Balance.

4. Describe Nicholson's Hydrometer, and explain how it may be used to determine the specific gravity of a liquid.

5. Is there any difference between the specific gravity and the density of a body? If so, explain precisely what that difference is.

6. Give a brief statement of the phenomena that are ascribed to Capillary Force; and explain, in particular, the physical cause by which mercury is depressed in a capillary tube.

7. If I place two pith balls in a basin of water, at a short distance from one another, they rush together; explain, as fully as you can, the cause of this phenomenon, and show how it may be traced to the influence of capillary force.

B. Exercises.

1. What is the weight of 48 cubic inches of copper, the specific gravity of which is 8·82? *Ans.* 245 ounces.

[Formula for the weight (W) of a body, in terms of its specific gravity (sp) and volume (V). If w be the weight of the unit volume of distilled water, then $W = V \times sp \times w$. In the British system it will sometimes be convenient to take the cubic foot as the unit volume, and then $w = 1000$ oz. $=$ 62·5 lbs.; but sometimes it will be more convenient to take the cubic inch as the unit volume, and then, $w = \frac{1000}{1728} = \cdot 5787$ oz. In the metric system the unit volume may be the cubic centimetre, and then $w =$ one gramme, or the cubic decimetre, and then $w =$ one kilogramme; in both cases, as w is equal to unity, it may be omitted from the formula, and $W = V \times sp$. To solve the problem before us, take the formula for the British system, adopting the cubic inch as the unit of volume; $W = V \times sp \times w = 48 \times 8{\cdot}82 \times \cdot 5787 =$ 245 oz.]

2. A block of marble, of specific gravity 2·7, weighs half a ton; find its volume. *Ans.* 6·637 cubic feet.

[Here it will be convenient to take the cubic foot as the unit of volume; and putting $W = 1120$ lbs., and $w = 62{\cdot}5$ lbs., we shall get the value of V in cubic feet. Thus, $W = V \times sp \times w \therefore V = \frac{W}{sp \times w} = \frac{1120}{2{\cdot}7 \times 62{\cdot}5} = 6{\cdot}637$.]

3. Find the volume of a block of stone the weight of which is 112 lbs., and the specific gravity 2·4. *Ans.* $\cdot 74\dot{6}$ of a cubic foot.

[$W = V \times sp \times w \therefore V = \frac{W}{sp \times w} = \frac{112}{2{\cdot}4 \times 62{\cdot}5} = \cdot 74\dot{6}$.]

4. Find the weight of 250 cubic centimetres of mercury, the specific gravity being 13·596. *Ans.* 3·399 kilogrammes.

[$W = V \times sp = 250 \times 13{\cdot}596 = 3399$ grammes.]

5. Find the weight of 200 cubic centimetres of brass, the specific gravity being 7·5. *Ans.* 1·5 kilogrammes.

6. A rectangular block of stone measures 86 × 37 × 16 centimetres and weighs 120 kilogrammes; find its specific gravity. *Ans.* 2·357.

[Here $V = 86 \times 37 \times 16 = 50{,}912$ cubic centimetres, or 50·512 cubic decimetres. Then taking $W = V \times sp$ we get $sp = \frac{W}{V} = \frac{120}{50{\cdot}512} = 2{\cdot}357$.]

7. A rectangular mass of masonwork measures 12 × 12 × 30 metres, and its specific gravity is 2·5; find its weight. *Ans.* 10,800,000 kilogrammes.

[The volume is here $12 \times 12 \times 30 = 4320$ cubic metres, or 4,320.000 cubic decimetres. Then taking $W = V \times sp$ we have $W = 4{,}320{,}000 \times 2{\cdot}5 =$ 10,800,000 kilogrammes, or 10,800 French tons.]

8. A cubic decimetre of lead, specific gravity 11·35, is wholly immersed in a vessel of alcohol, specific gravity ·8, and supported by a string; find the tension of the string. *Ans.* 10·55 kilogrammes.

[It is equal to the difference between the weight of the lead (W) and of the liquid displaced (W′). Now $W = V \times sp = 1 \times 11{\cdot}35 = 11{\cdot}35$ kilogrammes; and $W' = 1 \times {\cdot}8 = {\cdot}8$ of a kilogramme. Hence the tension of the string is $11{\cdot}35 - {\cdot}8 = 10{\cdot}55$.]

9. A homogeneous body floats in water with one-third of its volume immersed; what is its specific gravity? *Ans.* ${\cdot}\dot{3}$

10. A body weighs 58 grammes in air and 46 in distilled water; what is its specific gravity? *Ans.* $4{\cdot}8\dot{3}$

11. A specific gravity bottle holds 100 grammes of distilled water, and 180 grammes of sulphuric acid; what is the specific gravity of the sulphuric acid? *Ans.* 1·8.

12. A bottle filled with water weighs 212 grammes; 50 grammes of filings are thrown in, and the water that flows over is removed; the bottle remaining still full, is now found to weigh 254 grammes. Find the specific gravity of the filings. *Ans.* 6·25.

[The filings displace 8 grammes of water; therefore equal volumes of the filings and of the water weigh respectively 50 and 8 grammes; hence the specific gravity of the filings is $\frac{50}{8} = 6{\cdot}25$.]

13. A piece of glass, which weighs in air 13 grammes, is found to weigh in pure water 6·9 grammes, and in sea-water 6·2 grammes; find the specific gravity (1) of the glass, and (2) of the sea-water. *Ans.* (1) 2·13; (2) 1·1147.

[The glass loses, when immersed, a portion of its weight equal to the weight of the liquid displaced. But it loses in pure water $13 - 6{\cdot}9 = 6{\cdot}1$ grammes. Therefore equal volumes of the glass and of the water weigh respectively 13 and 6·1 grammes, and the specific gravity of the glass is $\frac{13}{6{\cdot}1} = 2{\cdot}13$. Again, the same piece of glass loses in sea-water $13 - 6{\cdot}2 = 6{\cdot}8$ grammes; therefore equal volumes of the sea-water and the pure water weigh respectively 6·8 and 6·1 grammes, and the specific gravity of the sea-water is $\frac{6{\cdot}8}{6{\cdot}1} = 1{\cdot}1147$.]

14. Calculate, in ounces, the greatest weight of Portland stone, specific gravity 2·5, that can be supported in water by 100 cubic inches of cork, specific gravity ·24. *Ans.* 73·3 ounces.

[The greatest weight will be supported when the Portland stone and the cork supporting it are both wholly immersed, and their joint weight just balanced by the upward pressure of the water. In these conditions, taking V for the volume of the Portland stone, we shall have the weight of the stone $= V \times 2{\cdot}5 \times w$, the weight of the cork $= 100 \times {\cdot}24 \times w$, and the upward pressure of the water $= V \times w + 100w$; and, putting the last equal to the sum of the former two, we have $V \times 2{\cdot}5 \times w + 100 \times {\cdot}24 \times w = V \times w + 100\,w$, or, dividing off by w, $V \times 2{\cdot}5 + 100 \times {\cdot}24 = V + 100$. Hence $V\,(2{\cdot}5 - 1) = 100\,(1 - {\cdot}24)$, and $V = \frac{76}{1{\cdot}5} = 50{\cdot}\dot{6}$ cubic inches; and the weight of the stone will be $W = V \times sp \times w = 50{\cdot}\dot{6} \times 2{\cdot}5 \times {\cdot}5787 = 73{\cdot}3$ ounces.]

15. A cube of lead, the side of which is 4 centimetres and the specific gravity 11·35, is just kept from sinking by a sphere of cork, specific gravity ·24, when both are wholly immersed in water; find the diameter of the sphere of cork. ***Ans.*** **11·8 centimetres.**

[Assuming r to represent the unknown radius of the sphere, calculate first the weight of the whole mass of lead and cork to be supported. The volume of the lead is $4^3 = 64$ cubic centimetres, and its weight is therefore $64 \times 11{\cdot}35 = 726{\cdot}4$ grammes; the volume of the cork is $\frac{4}{3}\pi r^3$, and its weight $\frac{4}{3}\pi r^3 \times {\cdot}24$. Hence the total weight of lead and cork is $\frac{4}{3}\pi r^3 \times {\cdot}24 + 726{\cdot}4$. Next calculate the upward pressure, which is equal to the weight of water displaced. This will be represented, in grammes, by the same figure that represents its volume in cubic centimetres; it will be, therefore, $\frac{4}{3}\pi r^3 + 64$. Now, from the conditions of the problem, the total upward pressure is equal to the total weight, since the one exactly balances the other, that is,

$$\frac{4}{3}\pi r^3 \times {\cdot}24 + 726{\cdot}4 = \frac{4}{3}\pi r^3 + 64.$$

$$\text{Hence } \frac{4}{3}\pi r^3\,(1 - {\cdot}24) = 726{\cdot}4 - 64,$$

$$\text{And } \frac{4}{3}\pi r^3 \times {\cdot}76 = 662{\cdot}4.$$

From this equation find $r^3 = 208$ nearly, and $r = 5{\cdot}9$; hence $2r$, the diameter required is 11·8 centimetres.]

MONTHLY EXAMINATION PAPER, No. 7.

PNEUMATICS, Section I.

A. Questions.

1. Give a brief account of the composition of the atmosphere.

2. Describe an experiment by which it may be proved that air has weight; and state what its weight is.

3. Explain fully and accurately what is meant by the pressure of the atmosphere, and point out the relation it has to the elastic force

of air; describe also some experiments by which your explanation may be illustrated.

4. Explain precisely what is meant by saying that the pressure of the atmosphere is equal to 755 millimetres, or that it is equal to 29 inches.

5. State briefly the principle of the mercurial Barometer; and give an historical sketch of the discovery and demonstration of this principle.

6. Explain the necessity for a correction for capacity, in the ordinary cistern barometer; give the formula to be employed in making this correction, and prove the truth of the formula.

7. Explain the method by which the necessity for a correction for capacity is obviated in Fortin's Barometer.

8. What is the use of the Vernier in connection with the barometer scale? Explain how it is graduated, and describe the way in which a reading is taken by its aid.

9. If a piece of glass rod happen to be floating in the tube of a barometer, will the mercury stand higher or lower in consequence? Give a full reason for your answer.

10. Explain, as fully as you can, the reasons why a balloon is usually inflated only to about three-fourths of its capacity, before beginning an ascent.

11. A leg of mutton weighs 10 lbs. when weighed in air; would it weigh more or less if weighed in a vacuum? How would you proceed to ascertain the difference without going through the process?

[The leg of mutton would weigh more in a vacuum. To prove this, and to find how we may determine the difference, let us take W to represent the downward pull of gravity on the leg of mutton, and p the upward pressure of the air against it. Its weight in air would evidently be $W - p$, while its weight in a vacuum would be W; the difference p is equal to the weight of the air displaced. This, however, is the *real* difference between the weight of the leg of mutton in air and its weight in a vacuum; and the question rather regards the *apparent* difference, that is, the difference that would be exhibited if we tested the weight by a pair of scales. To find this, let us take W' for the downward pull of gravity on the masses of metal employed to counterbalance the leg of mutton in air, and p' for the weight of air displaced by them. Since they balance the leg of mutton in air, it is evident that $W - p = W' - p' \therefore W = W' + (p - p')$. As p, the weight of the air displaced by the leg of mutton, is greater than p', the weight of the air displaced by the masses of metal employed to weigh it, it follows that $p - p'$ is a positive quantity, and therefore that W is greater than W'; and further, that the difference between them is equal to the excess of p above p'.]

12. What is a Water Hammer? Explain the way in which it is made, the phenomena it exhibits, and the cause of these phenomena.

B. Exercises.

1. A glass globe, 9·2 inches in diameter, weighs, when full of air, 846·6 grammes, and, when the air is exhausted, 838·2 grammes; find the weight of air in terms of cubic inches and grammes. *Ans.* 48·53 cubic inches of air weigh one gramme.

2. Find the height to which water may be raised in a suction pump, when the barometer stands at 760 millimetres, the specific gravity of mercury being 13·596. *Ans.* 1033·3 centimetres.

[The heights to which liquids will ascend in a vacuum, under a given pressure, are inversely proportional to their specific gravities. Hence if h and h', sp and sp' represent the heights and the specific gravities of the two liquids respectively, $\frac{h}{h'} = \frac{sp'}{sp}$ or $h \times sp = h' \times sp'$. This formula may be applied to the present exercise, because the water in the suction pump, and the mercury in the barometer tube, are both supported by the same pressure, namely, the pressure of the atmosphere. Hence taking h' for the height of the water and h for the height of the mercury, we have $h' = \frac{h \times sp}{sp'} = \frac{760 \times 13{\cdot}596}{1} = 10333$ millimetres or 1033·3 centimetres.]

3. When the barometer stands at 750 millimetres, find the pressure of the atmosphere on a square centimetre. *Ans.* 1·0197 kilogrammes.

[$W = 75 \times 13{\cdot}596 = 1019{\cdot}7$ grammes, or 1·0197 kilos.]

4. A pair of Magdeburg Hemispheres have been exhausted until the pressure of the air within is reduced to 15 millimetres of mercury; the diameter of the Hemispheres is 10 centimetres and the barometer stands at 755 millimetres. If the ring at one end of the apparatus be now hung on a hook attached to a fixed beam, what weight must be suspended from the ring at the other end to pull the Hemispheres asunder? *Ans.* 79·0432 kilogrammes.

[The pressure on each Hemisphere is equal to the weight of a column of mercury $75{\cdot}5 - 1{\cdot}5 = 74$ centimetres high, and having for base πr^2, r being the radius of the Hemispheres. Hence the force to be overcome is $\pi r^2 \times 74 \times 13{\cdot}6 = 3{\cdot}1416 \times 25 \times 74 \times 13{\cdot}6 = 79043$ grammes or 79·043 kilogrammes.]

5. The original Magdeburg Hemispheres, made by Otto von Guericke, were about 20 inches in diameter. Taking this estimate, calculate the force required to separate them, if the tension of the air

within were reduced to half an inch of mercury, and the barometer stood at 30·5 inches. *Ans.* 41 cwt., 1 qr., 16 lbs.

[Pressure to be overcome is, in ounces, $\pi r^2 \times 30 \times 13{\cdot}6 \times {\cdot}5781 = 314{\cdot}16 \times 30 \times 13{\cdot}6 \times {\cdot}5781 = 74177{\cdot}35$ oz. = 4636 lbs. = 41 cwt., 1 qr., 16 lbs.]

6. If a layer of water, 7 centimetres in height, be poured into the cistern of a barometer, will it increase the difference of level between the surfaces of mercury in the cistern and the tube; and if so, how much? *Ans.* 5·147 millimetres.

7. In a cistern barometer the cross section of the tube is ·5 of a square inch, the area of the cistern, outside of the tube, is 10 square inches, the neutral point is 30, the reading is 29·2; make the correction for capacity. *Ans.* The corrected reading is 29·16.

[If s represent the cross section of the tube, and S the area of the cistern outside the tube, and x the distance of the level of the mercury in the tube above or below the neutral point, the correction for capacity is $\frac{s}{S}x$, which is to be added to the reading, or subtracted from it, according as the reading is above or below the neutral point. The student should master the sense of this formula, and satisfy himself of its truth; otherwise it is not worth his while to apply it in working out exercises. In the case before us $\frac{s}{S}x = \frac{{\cdot}5}{10} \times {\cdot}8 = {\cdot}04$, and the correction is to be subtracted. Hence the corrected reading is $29{\cdot}2 - {\cdot}04 = 29{\cdot}16$.]

8. In a cistern barometer the diameter of the tube is ·5 of an inch, the cistern is a cylinder of 2 inches diameter, the reading is 29, the neutral point 30; find the corrected reading. *Ans.* 28·932.

[Cross section of tube, $\pi r^2 = 3{\cdot}1416 \times {\cdot}0625 = {\cdot}2$ nearly.
Area of cistern, $\pi r^2 = 3{\cdot}1416 \times 1$.
Area of cistern, outside of tube, $3{\cdot}1416 - {\cdot}2 = 2{\cdot}9416$.
Hence $\frac{s}{S}x = \frac{{\cdot}2}{2{\cdot}9416} \times 1 = {\cdot}068$; and the corrected reading will be $29 - {\cdot}068 = 28{\cdot}932$.]

9. A cistern barometer is lowered into a vessel of water until the level of mercury in the cistern is 6 inches below the surface of the water; what will be the change in the reading of the barometer? *Ans.* An increase of ·441 of an inch.

[The increase of height in the barometric column will balance the increase of pressure caused by 6 inches of water. In this case evidently the heights are inversely proportional to the specific gravities, that is $h \times sp = h' \times sp'$; hence, taking h for the increase in the barometric column, we have $h = \frac{h' \times sp'}{sp} = \frac{6 \times 1}{13{\cdot}6} = {\cdot}441$.]

10. A traveller ascends from the plain, where the mercury in the barometer stands at 30 inches, to the top of a mountain, about 12,000 feet high, where it stands at 20 inches. Find the change which the pressure of the atmosphere on his body has undergone, during the ascent. *Ans.* It has been reduced from 2125 lbs. to 1416·$\dot{6}$ lbs. on every square foot of surface.

11. A spherical balloon, 4 metres in diameter, made of oil silk that weighs ·25 kilos. per square metre, is filled with hydrogen; calculate its lifting power when the weight of air is 1·3 kilos., and of hydrogen ·09 of a kilo., per cubic metre. *Ans.* 27·981 kilos.

[The lifting power is the upward pressure less by the downward pressure. The upward pressure is equal to the weight of the air displaced; the downward pressure is equal to the weight of the hydrogen plus the weight of the oil silk. To get the weight of the air displaced, we must get the volume of the balloon, in cubic metres, and multiply it by the weight of one cubic metre of air; that is,

$$\frac{4}{3}\pi r^3 \times 1{\cdot}3 = 33{\cdot}51 \times 1{\cdot}3 = 43{\cdot}563 \text{ kilos.}$$

Likewise the weight of the hydrogen will be the volume of the balloon, in cubic metres, multiplied by the weight of one cubic metre of hydrogen,

$$\frac{4}{3}\pi r^3 \times {\cdot}09 = 33{\cdot}51 \times {\cdot}09 = 3{\cdot}016 \text{ kilos.}$$

The weight of the oil silk will be equal to the spherical surface of the balloon, in square metres, multiplied by the weight of one square metre of oil silk,

$$4\pi r^2 \times {\cdot}25 = 50{\cdot}265 \times {\cdot}25 = 12{\cdot}566 \text{ kilos.}$$

Hence the lifting power is

$$\frac{4}{3}\pi r^3 \times 1{\cdot}3 - \left(\frac{4}{3}\pi r^3 \times {\cdot}09 + 4\,\pi r^2 \times {\cdot}25\right) = 43{\cdot}563 - (3{\cdot}016 + 12{\cdot}566) = 27{\cdot}981 \text{ kilos.]}$$

12. Giffard's Captive Balloon, exhibited in Paris in the summer of 1878, was 36 metres in diameter; and the material of which it was composed weighed one kilogramme per square metre. Find its lifting power, if fully inflated; the weight of air may be taken at 1·3, and of hydrogen at ·09 per cubic metre. *Ans.* 25,487·6 kilos.

[The weight of the air displaced would be,

$$\frac{4}{3}\pi r^3 \times 1{\cdot}3 = 31{,}757{\cdot}7 \text{ kilos.}$$

The weight of the hydrogen,

$$\frac{4}{3}\pi r^3 \times {\cdot}09 = 2{,}198{\cdot}6 \text{ kilos.}$$

The weight of the material,

$$4\pi r^2 \times 1 = 4{,}075{\cdot}5 \text{ kilos.}$$

Hence the lifting power would be

$$\frac{4}{3}\pi r^3 \times 1{\cdot}3 - \left(\frac{4}{3}\pi r^3 \times {\cdot}09 + 4\,\pi r^2\right) = 31{,}757{\cdot}7 - (2{,}198{\cdot}6 + 4071{\cdot}5) = 25{,}487{\cdot}6 \text{ kilos.]}$$

MONTHLY EXAMINATION PAPER, No. 8.

PNEUMATICS, SECTION II.

A. Questions.

1. Explain Boyle's law ; write down the several formulas in which it may be expressed, and point out their relations to one another.

2. Describe the experiments with Mariotte's tube, by which Boyle's law may be proved; and state under what restrictions this law is to be admitted.

3. Describe the open air mercurial pressure gauge, or manometer, and explain the principle of its action.

4. Explain the construction and action of the Air Gun. Prove that if H represent the pressure of the atmosphere, V the volume of the air chamber, and v the volume of the condenser, the pressure, after n strokes, (P_n), will be represented by the formula $P_n = \frac{V + nv}{V} H$.

5. Describe and explain the Suction Pump; and show to what height it can pump up water, and to what height it could pump up mercury.

6. Prove that, in the suction pump, after it has commenced discharging water, the pressure to be overcome, in raising the piston, is equal to the weight of a column of water whose base is the cross section of the piston, and whose height is the height of the spout of the pump above the level of the water in the well.

7. Explain and prove the formula that expresses the exhaustion produced by a theoretically perfect air pump, $T_n = \left(\frac{V}{V + v}\right)^n \times H$, T_n being the tension of the air at the end of any number of strokes n, V the volume of the receiver, together with the connecting pipe, v the volume of the barrel of the Air Pump below the highest level of the piston, and H the pressure of the atmosphere.

8. Explain and prove the formula $T = H \frac{v'}{v}$, which expresses the limit of the exhausting power of a theoretically perfect air pump, T being the tension when that limit is reached, H the pressure of the atmosphere, v the volume of the barrel, and v' the space not traversed by the piston.

9. Explain the construction and the principle of the Diving Bell.

B. Exercises.

1. If a certain quantity of gas has a volume of 340 cubic inches, under a pressure of 21 inches of mercury, what will be its volume under a pressure of 30 inches? *Ans.* 238 cubic inches.

[Formula for the relations of the volume (V), pressure (P), and density (D) of a gas, the temperature remaining constant; $\frac{V}{V'} = \frac{P'}{P} = \frac{D'}{D}$, or $VP = V'P' = V'D'$. Hence, in the present case, $V' = \frac{VP}{P'} = \frac{340 \times 21}{30} = 238$.]

2. A given quantity of a gas has a volume of 8 litres, under a pressure of 480 millimetres; find what its volume will be under a pressure of 640 millimetres. *Ans.* 6 litres.

$$[V' = \frac{VP}{P'} = \frac{8 \times 480}{640} = 6.]$$

3. If a cubic foot of air, under a pressure of one atmosphere, were condensed into a cubic inch, what would be the tension of the condensed air, in atmospheres? *Ans.* 1728.

[The tension of the condensed air would just balance the compressing force and the compressing force is given by the formula $P' = \frac{VP}{V'} = \frac{1728 \times 1}{1}$.]

4. A faulty barometer has some air above the mercury in the tube, in consequence of which the mercury stands, on a certain day, at 29 inches, when that of a correct barometer stands at 30 inches; what is the tension of the air above the mercury, and what is its density as compared with the atmosphere? *Ans.* (1) one inch of mercury; (2) $\frac{1}{30}$ of the atmosphere.

5. The volume of air in the short limb of a Mariotte's tube is 20 cubic centimetres, under the pressure of the atmosphere, on a day that the barometer stands at 750 millimetres; what will it be if I pour in mercury into the long limb until the level of the mercury in the long limb is 450 millimetres higher than in the short limb? *Ans.* 12·5 cubic centimetres.

$$[V' = \frac{VP}{P'} = \frac{20 \times 750}{750 + 450} = 12{\cdot}5.]$$

6. A spherical bubble of air has a diameter of half an inch at a depth of 5 feet below the surface of a lake; find at what depth the diameter would be reduced to a quarter of an inch. The height of the barometer may be taken as 30 inches. *Ans.* 278 feet.

[The volumes of spheres are related as the cubes of their diameters; therefore when the diameter is reduced to one-half, the volume will be reduced to one-eighth. Again, the pressure at a depth of 5 feet is the pressure of the atmosphere plus the pressure of the water, which may be expressed in terms of water pressure, by $2{\cdot}5 \times 13{\cdot}6 + 5 = 39$ feet of water. Now to reduce the volume eight fold, this pressure must be increased eight fold, in other words, the increase of depth must be $39 \times 7 = 273$ feet, and the total depth 278.]

7. I pour mercury into a barometer tube until the length of tube occupied by air is reduced to 15 centimetres; then pressing my finger on the open end, I invert the tube, and, without letting any more air enter, I plunge it in a vessel of mercury. The mercury in the tube falls until the length of tube occupied by the air is 25 centimetres, and it then stands at a height of 302 millimetres above the level of the mercury in the cistern. Find the pressure of the atmosphere. *Ans.* 755 millimetres.

[The pressure of the atmosphere evidently balances 302 millimetres of mercury plus the tension of the air in the tube. This air first existed at the pressure of the atmosphere (P), and occupied a length of 15 centimetres; it then expanded to 25 centimetres and acquired its new tension (P′); the volume has therefore increased in the ratio of 15 : 25, and $\frac{V}{V'} = \frac{15}{25}$. But $P' = \frac{V\,P}{V'}$ $\therefore$ $P' = \frac{15}{25}P = \frac{3}{5}P$; and this pressure, plus 302 millimetres, balances the pressure of the atmosphere, which is P, that is, $\frac{3}{5}P + 302 = P$. Hence $\frac{2}{5}P = 302$ and $P = 302 \times \frac{5}{2} = 755$.]

8. A glass globe, containing air at a pressure of 770 millimetres, is connected by a stopcock with the Torricellian Vacuum of a barometer constructed on Fortin's principle. The reading of the barometer is 760 millimetres, the length of the Torricellian Vacuum 100 millimetres, and the cross section of the tube 2 square centimetres. On the stopcock being turned, the air enters from the glass vessel, and the mercury sinks in the tube until the reading is 100 millimetres. Find the volume of the glass vessel. *Ans.* 912 cubic centimetres.

[Let x represent the volume in cubic centimetres. It appears that air, having a volume x with a pressure of 77 centimetres, expands, after the stopcock is turned, until its volume becomes $x + 2\,(10 + 66)$ cubic centimetres, and its pressure is then $76 - 10 = 66$ centimetres of mercury. Calling the new volume V′, and taking the formula $V' = \frac{V\,P}{P'}$ we have $x + 2\,(10 + 66) = \frac{x \times 77}{76 - 10}$, from which $x = 912$.]

9. A cylindrical glass tube, 2 metres in length and 6 centimetres in diameter, closed at one end, and provided with a stopcock at the other, is screwed to the plate of an air pump. The barrels of the air pump are 30 centimetres long and 8 centimetres in diameter; and the small tube connecting them with the plate has an internal volume

of 4 cubic centimetres. When the pump has been worked for some time, the tension of the air in the barrels is reduced to 10 millimetres; the stopcock is then turned, and the air from the cylindrical glass tube is allowed to mix freely with the air in one barrel of the air pump. Find the tension of the mixture. *Ans.* 600·1 millimetres.

[Formula for the mixture of gases. If two gases, whose volumes and pressures are V_1 P_1, and V_2 P_2, respectively, are introduced into a space whose volume is $\overline{V}$, then the pressure $\overline{P}$ will be got by the formula $\overline{V}\,\overline{P} = V_1 P_1 + V_2 P_2$, and $\overline{P} = \frac{V_1 P_1 + V_2 P_2}{\overline{V}}$. In the case before us, the two gases are the air in the tube, at a pressure of 760 millimetres ($V_1 P_1$), and the air in the barrel and connecting tube, at a pressure of 10 millimetres ($V_2 P_2$). Now V_1 the volume of the tube is $\pi r^2 \times 200 = 3{\cdot}1416 \times 9 \times 200 = 5654{\cdot}9$ cubic centimetres, and V_2, the volume of the barrel plus the connecting tube, is $\pi r^2 \times 30 + 4 = 3{\cdot}1416 \times 16 \times 30 + 4 = 1512$ cubic centimetres, while $\overline{V}$, which is the sum of both, is $5654{\cdot}9 + 1512 = 7166{\cdot}9$ cubic centimetres. Thus we have $\overline{P} = \frac{V_1 P_1 + V_2 P_2}{\overline{V}} = \frac{5654{\cdot}9 \times 760 + 1512 \times 10}{7166{\cdot}9} = 600{\cdot}1$.]

9. The receiver of a theoretically perfect air pump, is 5 times as large as the barrel; find the tension of the air after 3 strokes. *Ans.* ·578 of an atmosphere.

[Formula for the degree of exhaustion produced by an air pump, in n strokes, $T_n = \left(\frac{V}{V + v}\right)^n \times H$, which becomes in the present case $T_3 = \left(\frac{V}{V + \frac{V}{5}}\right)^3 \times H = \left(\frac{V}{V(1 + \frac{1}{5})}\right)^3 \times H = \left(\frac{5}{6}\right)^3 \times H = {\cdot}578\,H$.]

10. In a theoretically perfect air pump what is the tension, in millimetres of mercury, after 10 strokes, if the barrel is two-thirds the size of the receiver, and the pressure of the atmosphere 750 millimetres. *Ans.* 4·535 millimetres.

[$T_{10} = \left(\frac{V}{V + v}\right)^{10} \times 750 = \left(\frac{V}{V + \frac{2}{3}V}\right)^{10} \times 750 = \left(\frac{3}{5}\right)^{10} \times 750 = {\cdot}00604665 \times 750 = 4{\cdot}535$.]

11. What must be the capacity of the barrel of an air pump, if the air in a receiver of 4 litres capacity is to be reduced to one-third of its tension in 2 strokes. *Ans.* 2·928 litres.

[$T_2 = \left(\frac{V}{V + v}\right)^2 \times H = \frac{4^2}{(4 + v)^2} \times H = \frac{16H}{(4 + v)^2}$. But from the conditions of the problem $T_2 = \frac{H}{3}$, $\therefore \frac{H}{3} = \frac{16H}{(4 + v)^2}$, from which $v = 2{\cdot}928$.]

12. Give the tension of the air, in terms of the pressure of an atmosphere, after each of the first four strokes, when the barrel is one-fourth the size of the receiver. *Ans.* (1) ·8, (2) ·64, (3) ·512, (4) ·4099.

13. Find the pressure of the air in the air chamber of an air gun, after 40 strokes of the condenser, when the volume of the condenser is one-fifth the volume of the air chamber. *Ans.* Equal to 9 atmospheres.

[Formula for the pressure of the air in an air gun, after n strokes, $P_n = \frac{V + nv}{V} \times H$, V being the volume of the air chamber, v the volume of the condenser, and H the pressure of the atmosphere. In the exercise before us, this formula becomes, $P_{40} = \frac{V + 40\frac{V}{5}}{V} \times H = 9H$.]

14. How many strokes would be necessary to produce a pressure of 8 atmospheres in the air chamber of an air gun, if the volume of the condenser be one-sixth of the volume of the air chamber? *Ans.* 42.

[$P_n = \frac{V + nv}{V} \times H$, that is, in the case before us, $8\,H = \frac{V + n\frac{V}{6}}{V} \times H = \left(1 + \frac{n}{6}\right) \times H \therefore 8 = 1 + \frac{n}{6}$ and $n = 42$.]

15. What must be the ratio of the volume of the condenser to the volume of the air chamber, in order to produce a pressure of 20 atmospheres in 50 strokes? *Ans.* $\frac{v}{V} = \cdot 38$.

[Let x be the ratio of v to V; then $\frac{v}{V} = x$, and $v = xV$. Next, taking the formula $P_n = \frac{V + nv}{V} \times H$, and applying it to the conditions of the problem, we get, $20\,H = \frac{V + 50xV}{V} \times H$, or $20 = 1 + 50x$, and $x = \frac{19}{50} = \cdot 38$.]

16. The air chamber has 10 times the volume of the condenser. Find the number of strokes necessary to produce a pressure of 30 atmospheres. *Ans.* 190.

[$20\,H = \frac{V + n\frac{V}{10}}{V} \times H = \left(1 + \frac{n}{10}\right)H \therefore 20 = 1 + \frac{n}{10}$, and $n = 190$.]

17. If I want to produce a pressure of 15 atmospheres in 20 strokes, what must be the ratio of the volume of the condenser to the volume of the air chamber? *Ans.* $\frac{v}{V} = \cdot 75$.

[Applying the formula $P_n = \frac{V + nv}{V} \times H$, we get $15\,H = \frac{V + nxV}{V} \times H = 20x\,H$; from which $x = \frac{15}{20} = \cdot 75$.]

GENERAL EXAMINATION PAPER IN MECHANICS.

A. Questions.

1. Explain what you mean by the word *force*, and illustrate your explanation by reference to some of the forces of Nature.

2. Explain clearly and fully what is meant by the Composition and Resolution of forces.

3. Apply the doctrine of the resolution of forces to the case of a sailing vessel, and illustrate your explanation by a diagram.

4. Explain and prove the method by which the true weight of a body may be ascertained with a false balance. (London University, Matriculation, Jan. 1876).

5. State and prove the condition of equilibrium in a combination of wheels and axles, neglecting friction.

6. Describe and explain the experiment known as the Schehallien experiment, by which the law of gravitation has been confirmed.

7. State fully the reasons why the force of gravity varies with the latitude; and write down the exact value of the force of gravity at three or four important places in different latitudes.

8. State exactly what is meant when it is said that gravity in Edinburgh is 32·2, or $g = 32{\cdot}2$. (London University, Matriculation, Jan. 1876).

[The formula $g = 32{\cdot}2$ is employed in two senses, which, though closely allied, are nevertheless quite distinct from one another. It means that the *velocity generated* by gravity when it acts on a body for a second is 32·2 feet a second. It also means that the *force* of gravity, acting on a unit mass, is equal to 32·2 British kinetic *units of force*. Hence the formula $g = 32{\cdot}2$ is really ambiguous, and if we wish to speak clearly we should say: The *velocity generated* by gravity acting on any body for a second is 32·2 *feet*; or else, the *force of gravity* acting on a unit mass is equal to 32·2 British kinetic units.]

9. Suppose that a straight tube, two or three feet in diameter, were thrust down into the earth, at the equator, in the direction of the centre of gravity of the earth, and that a stone were dropped down the centre of the tube; what direction would the stone take? Explain your answer fully.

10. Give the principal formulas that express the relations of velocity, time, and space, in the case of a body falling vertically, under the action of the force of gravity.

11. A body is moving in a circle with uniform velocity; show that it must be acted on by a constant force, the direction of which is *towards* the centre of the circle; and find the magnitude of the force in terms of the velocity and mass of the body, and the radius of the circle, $f = \frac{mv^2}{r}$.

12. Explain the construction and use of Kater's pendulum.

13. Define what is meant by Work, and state how it may be measured. How would you convert Kilogrammetres into foot pounds?

14. Is there any sense in which a Perpetual Motion machine may be conceived possible? If so, explain the principle on which such a machine may be conceived to be constructed, and show that it is not liable to the objection commonly urged against the possibility of Perpetual Motion.

15. Give some account of the theories that have been propounded to account for the maintenance of the Sun's energy.

16. Explain Sir William Thomson's theory of the Dissipation of Energy.

B. Exercises.

1. Three cords are tied together at a point. One of them is pulled in a northerly direction with a force of 6 lbs., and another in an easterly direction with a force of 8 lbs. With what force must the third cord be pulled in order to keep the whole at rest? (London University, Matriculation, June, 1876.) *Ans.* 10 lbs.

2. Two forces, of 9 and 11 lbs. respectively, act at an angle of 120°; find the resultant. *Ans.* 10·148 lbs.

[$R^2 = P^2 + Q^2 + 2PQ\cos\theta = 9^2 + (11)^2 + 2 \times 99 \times \cos 120° = 81 + 121 - 2 \times 99 \times \cdot 5 = 202 - 99 = 103 \therefore R = \sqrt{103} = 10{\cdot}148.$]

3. A ball weighing 10 lbs., which slides along a smooth rod inclined at an angle of 45° to the horizon, is kept at rest by a force which acts along the rod; find the magnitude of the force, and the pressure on the rod. *Ans.* (1) 7·071 lbs.; (2) 7·071 lbs.

[Draw a diagram representing the conditions of the problem; observe that the ball is kept at rest by the action of three forces, namely, its own weight ($W = 10$ lbs.), the applied force (P), and the reaction of the rod (R), which is equal to the pressure on it, and that these three forces may be represented by the sides of a right angled triangle. From the diagram it will be apparent that $R = P$; also that $W^2 = P^2 + R^2 = 2P^2 \therefore W = \sqrt{2}\,P$, and $P = \frac{W}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7{\cdot}071$ lbs.]

4. To one corner of a homogeneous square plate is attached a weight equal to half the weight of the plate; find the centre of gravity of the whole.

[Draw a diagram representing the plate, with the weight attached at one corner. Suppose the whole plate to be concentrated at its centre of gravity, which is at the intersection of its two diagonals; then show that the centre of gravity of the plate, so concentrated, and the weight at the corner, will be on the straight line joining these two points, at a distance from the corner equal to two-thirds of the line.]

5. A pound weight is attached to the vertex of a homogeneous triangular plate, which itself weighs a pound; find the centre of gravity of the triangle and the pound weight taken together.

[Proceed as in the last exercise, and find the required centre of gravity in the line joining the vertex and the middle point of the base, at a distance from the vertex equal to one third of the line.]

6. Squares are described upon the three sides of an isosceles right angled triangle. Determine the centre of gravity of the complete figure so formed. (London University, Matriculation, Jan. 1876.) *Ans.* In the median line of the triangle, at a distance of $\frac{1}{27}$ from the base.

7. A weight of 48 lbs. is supported by two strings, which are respectively 3 feet and 4 feet long, and are fastened to points in the same horizontal line, at such distances apart that the strings make right angles with one another; find the tension of each string. *Ans.* 38·4 lbs., and 28·8 lbs.

[Draw a diagram representing the conditions of the problem. Observe that the two strings and the horizontal line constitute a right angled triangle, the sides of which are respectively perpendicular to the directions in which the three forces act, that keep the body at rest; therefore the sides of the triangle represent the magnitudes of the forces respectively. Next show that the hypothenuse, which represents the weight, is to the small sides in the ratio 5 : 4 : 3; and it follows that, if W, P, Q represent the weight and the tensions of the strings, $W:P:Q::5:4:3$. Hence $P=\frac{4W}{5}=\frac{192}{5}=38{\cdot}4$, and $Q=\frac{3W}{5}=\frac{144}{5}=28{\cdot}8$.]

8. A nut is placed in a nutcrackers, at a distance of $\frac{3}{4}$ of an inch from the joint, and a force of 5 lbs., applied at 4 inches from the joint, is just sufficient to break it; find the resistance offered by the nut. *Ans.* $26\frac{2}{3}$ lbs.

9. A weight of 20 lbs. rests on a smooth inclined plane, which rises 3 in 5; find the force which will be just sufficient, if applied horizontally, to support the weight. *Ans.* 15 lbs.

10. A stone projected vertically upwards, from the surface of the earth, reached the ground again in six seconds. Neglecting the resistance of the air, find what its height was at the end of the first second. (London University, Matriculation, June, 1876.) *Ans.* 80 feet.

11. A projectile, thrown vertically upwards, reaches a height of 510·22 metres before it comes to rest; find the velocity of projection, neglecting the resistance of the air. *Ans.* 100 metres.

[In the metric system g may be taken equal to 9·8 metres; and the velocity of projection, which carries a body to a height of 510·22 metres, is the same as would be acquired in falling through that distance. To find this velocity take the formula $V^2 = 2gS = 2 \times 9{\cdot}8 \times 510{\cdot}22 = 10{,}000{\cdot}312$, and we get $V = \sqrt{10{,}000{\cdot}312} = 100$ nearly.]

12. Find the time a body will take to fall through 2,000 metres, and the velocity it will acquire in falling, neglecting the resistance of the air. *Ans.* (1) 20·203 seconds; (2) 196·196 metres.

[$S = \frac{1}{2}gt^2 = 4{\cdot}9 \times t^2 \therefore t^2 = \frac{2000}{4{\cdot}9} = 408{\cdot}1632$, and $t = 20{\cdot}203$. Again, $V = gt = 9{\cdot}8 \times 20{\cdot}2 = 196{\cdot}196$.]

13. A stone is thrown vertically down into a pit 150 metres deep, and reaches the bottom in 4 seconds; find the velocity with which it was projected, and the velocity it has when it reaches the bottom. *Ans.* (1) 17·9 metres; (2) 57·1 metres.

[$S = V't + \frac{1}{2}gt^2 = 4\,V' + 4{\cdot}9 \times 16$. Hence $150 = 4\,V' + 78{\cdot}4$, and $4\,V' = 71{\cdot}6$, and $V' = 17{\cdot}9$. Again $V = V' + gt = 17{\cdot}9 + 9{\cdot}8 \times 4 = 57{\cdot}1$.]

14. Find the time of oscillation of a pendulum whose length is ·99384 in a place where gravity is 9·81. *Ans.* One second.

$$[t = \pi\sqrt{\frac{l}{g}} = 3{\cdot}1416 \times \sqrt{\frac{{\cdot}99384}{9{\cdot}81}} = 1.]$$

15. A pendulum whose length is 1·95 metres, makes 61,682 oscillations in a day, in the latitude of Paris; find the length of a seconds pendulum for the same latitude. *Ans.* ·99385 metres.

[From the formulas of the pendulum, if l be the length of any pendulum, t the time of an oscillation, and L the length of a seconds pendulum, we have $l = \frac{gt^2}{\pi^2}$; from which we get $L = \frac{g}{\pi^2} = \frac{l}{t^2}$. In the present case $l = 1{\cdot}95$ and $t = \frac{61{,}682}{86{,}400}$, since there are 86,400 seconds in a day; therefore $L = 1{\cdot}95 \div \left(\frac{61{,}682}{86{,}400}\right)^2 = {\cdot}99385$.]

16. At each end of a rod, 8 feet long, is fixed a ball of 10 lbs. weight, and the rod revolves 100 times a minute, round a vertical

axis at its centre; find the tension of the rod in lbs. The weight of the rod may be neglected. *Ans.* 137·08 lbs.

[The tension of the rod will be equal to the centrifugal force of either ball. Find the periodic time to be ·6 of a second. Then $F = \frac{W}{g} \times \frac{4\pi^2 r}{T^2} = \frac{10}{32} \times \frac{4 \times 9{\cdot}87 \times 4}{{\cdot}36} = \frac{987}{7{\cdot}2} = 137{\cdot}08$.]

17. Find what the rate of rotation of the earth should be, in order that bodies at the equator should have no weight. *Ans.* The rate of rotation should be increased 17 fold.

[The total force of gravity at the equator is 289 times greater than centrifugal force at the equator, say $G = 289f$. But $f = \frac{v^2}{r}$; therefore, if v^2 were 289 times as great as it is, f would be exactly equal to G, and would balance it, as it acts in the opposite direction: in this condition of things it is evident that bodies at the equator would have no weight. If we call the increased velocity v', the force of gravity would be balanced by centrifugal force when $v'^2 = 289\, v^2$, that is, when $v' = 17v$.]

18. A projectile weighing 2,000 lbs. has been discharged from the 100-ton gun at Woolwich, with a velocity of 1,600 feet a second; find its kinetic energy in foot tons, on leaving the gun. *Ans.* 35,680.

[Kinetic energy $= \frac{Wv^2}{2g} = \frac{2000}{64} \times (1600)^2 = 79{,}923{,}200$ foot lbs. = 35,680 foot tons.]

19. A railway truck, weighing 12 tons, is drawn from rest by a horse along a level railway, and, after having passed over a space of 50 feet, is moving at the rate of 3 miles an hour; find how many units of work the horse has done up to that point, the resistance of friction and the air being 8 lbs. per ton. *Ans.* 12,931·2 foot pounds.

[The work done is two-fold: first, the resistance is overcome through a space of 50 feet; this is equal to $12 \times 8 \times 50 = 4800$ foot pounds; secondly, a velocity of 3 miles an hour has been imparted to a mass of 12 tons; this work is exactly equal to the kinetic energy of the moving mass, which is $\frac{Wv^2}{2g} = \frac{12 \times 2240 \times (4{\cdot}4)^2}{2 \times 32} = 8131{\cdot}2$ foot pounds. The total work done by the horse is, therefore, $4800 + 8131{\cdot}2 = 12931{\cdot}2$ foot pounds.]

20. A tramcar, weighing, with its passengers, 4 tons, and moving at the rate of 5 miles an hour, on a level road, is stopped; find what amount of work must be done by the horses simply to restore the velocity destroyed. *Ans.* 3·3611 foot tons.

[The work to be done is precisely to restore the kinetic energy of the moving mass. The kinetic energy of the tramcar moving at the rate of 5 miles an hour is given by the formula $\frac{Wv^2}{2g}$; which becomes, if we take W in tons, and v in feet per second, $\frac{4}{64} \times (7\frac{1}{3})^2 = \frac{53\cdot7}{16} = 3\cdot3611$ foot tons.]

21. Suppose a tramcar, weighing 4 tons, to be drawn 4 miles on a level road, the resistance of friction and of the air being 10 lbs. a ton; suppose further, that it is stopped 12 times on the way, and, after each stoppage, is gradually brought to a maximum velocity of 5 miles an hour; required to find (1) what is the amount of work necessary to draw the car over the distance, and (2) what is the additional work occasioned by the stoppages. *Ans.* (1) 377·1 foot tons; (2) 40·3332 foot tons.

[The work necessary to draw the car 4 miles is equal to the resistance multiplied by the distance through which it is overcome; that is, $40 \times 4 \times 1760 \times 3 = 844{,}800$ foot pounds, or 377·1 foot tons. The additional work occasioned by each stoppage, is simply to generate a velocity of 5 miles an hour in a mass of 4 tons weight; and, in the last exercise, it was shown that this requires 3·3611 foot tons of work. Hence the 12 stoppages occasion an amount of work equal to $3\cdot3611 \times 12 = 40\cdot3332$ foot tons.]

GENERAL EXAMINATION PAPER IN HYDROSTATICS AND PNEUMATICS.

A. Questions.

1. Prove that the pressure of a liquid at any point of the surface of a body, with which it is in contact, is perpendicular to the surface of the body at that point.

2. Prove that the pressure of a liquid, at rest, is the same at all points of the same horizontal plane.

3. Show that, notwithstanding the great mechanical advantage of the Hydrostatic Press, there is always an equality between the work done and the energy expended.

4. Show that the pressure on the bottom of a vessel may be much greater, or much less, than the weight of the liquid it contains, and illustrate your explanation with diagrams.

5. Take the case of a vessel, full of water, so shaped that the pressure on the base is only equal to one-fourth of the weight of the water contained, and show how it is that, if the vessel be placed on

the pan of a balance, the pan will be pressed down with a force equal to the *whole* weight of the water contained, plus the weight of the vessel.

6. Describe the Spirit Level, and explain the principle on which it acts.

7. Explain the principle of the Universal Hydrometer, of variable volume, and the method of graduating it.

8. Though copper is nearly nine times as heavy as water, yet if I take short lengths of thin copper wire, and pass them once or twice between my fingers, they will float on the surface of water; explain the physical cause of this phenomenon.

9. Who was the first to prove that air has weight, and how did he prove it? Describe the experiment made for the purpose by Aristotle, and explain the cause of his failure.

10. Describe the cistern of Fortin's barometer; explain how it is to be used; and point out its advantages.

11. Explain the construction and action of the condenser; and prove that, if H represent the pressure of the atmosphere, v the volume of the condenser, and v' the volume of the space not traversed by the piston, then the limit of condensing power is reached when the tension of the condensed air is equal to $H\frac{v}{v'}$.

12. Explain some form of air pump, and show how the degree of exhaustion produced by n strokes may be computed.

13. Explain the construction and principle of the siphon, illustrating your explanation with a diagram.

14. If a diving bell is lowered to a depth of 51 feet, state roughly to what degree the air within it is condensed.

B. Exercises.

1. The diameter of the large piston in a hydrostatic press is 25 inches, of the small piston 2 inches; the lever is 30 inches long, and is attached to the small piston rod at a point 2 inches from the fulcrum; what resistance will be overcome by a power of 56 lbs. applied at the end of the lever? *Ans.* 58·594 tons.

[The mechanical advantage of the lever is 15; therefore the power applied to the small piston is 56 × 15 = 840 lbs. Taking then the formula, $P = p\frac{S}{s} = p\frac{D^2}{d^2}$, we get $P = 840 \times \frac{625}{4} = 210 \times 625 = 131{,}250$ lbs. or 58·594 tons.]

2. In a hydrostatic bellows, the height of the tube, above the highest level of the bellows when full, is 182 centimetres, its diameter is 1·3 centimetres, the diameter of the top board of the bellows is 30 centimetres; find the weight that the bellows is capable of lifting. *Ans.* 128·6 kilogrammes.

[$P = p\frac{D^2}{d^2} = p \times \frac{(30)^2}{(1{\cdot}3)^2} = p \times 532{\cdot}5$. To find p we have simply to get the weight of a cylindrical column of water, 182 centimetres high, and 1·3 centimetres in diameter. Taking r for the radius and h for the height of the column, we get its volume $\pi r^2 \times h = 3{\cdot}14159 \times {\cdot}4225 \times 182 = 241{\cdot}514$ cubic centimetres, and its weight 241·514 grammes. Hence $P = 241{\cdot}514 \times 532{\cdot}5 = 128{,}600$ grammes, or 128·6 kilogrammes.]

3. State roughly what is the pressure on the body of a fish at a depth of 102 feet below the surface of a lake. *Ans.* About four atmospheres, or 60 lbs. on each square inch of surface.

4. A house is supplied with water, by means of a pipe from a reservoir, the surface of which is 241 feet above the street level; find the pressure, per square inch, on a tap 25 feet above the level of the street. The weight of water may be taken as 62·5 lbs. per cubic foot. *Ans.* 93·75 lbs.

[It is equal to the weight of a column of water one square inch in section, and $241 - 25 = 216$ feet high; that is $216 \times \frac{1}{144} \times 62{\cdot}5 = 93{\cdot}75$.]

5. A vessel having the shape of a pyramid, the base of which is a square, is filled with water; measured internally each side of the base is 4 inches, and the altitude of the vessel is 5 inches; find the pressure on the base, and the weight of the water contained. *Ans.* (1) 46·296 oz.; (2) 15·423 oz.

[The pressure on the base is equal to the weight of a column of water, 5 inches high and 16 square inches in cross section; that is $16 \times 5 \times {\cdot}5787 = 46{\cdot}296$ oz. The weight of the water contained is equal to the volume in cubic inches, multiplied by the weight of a cubic inch of water; that is, $\frac{1}{3} \times 16 \times 5 \times {\cdot}5787 = 15{\cdot}432$ oz.]

6. If the vessel, in the last exercise, be placed on the pan of a balance, it will be found that the pressure on the pan is hardly more than one-third of the pressure on the bottom of the vessel; in fact, it exceeds that amount only by the weight of the vessel itself. What then becomes of the remaining two-thirds of the pressure on the base?

[The pressure of the water on the lateral surfaces of the pyramid is, at all points, perpendicular to these surfaces; at every point the pressure may be resolved into two components, one horizontal and outwards, the other vertical and upwards; the horizontal components balance one another, and the vertical components remain to balance two-thirds of the pressure on the base.]

7. A cylindrical glass jar, 8 centimetres in diameter, is filled with mercury to a height of 10 centimetres; find the pressure on the bottom, the specific gravity of mercury being 13·6. *Ans.* 6·836 kilogrammes.

[$W = V \times sp = \pi r^2 \times 10 \times 13{\cdot}6 = 3{\cdot}1416 \times 16 \times 10 \times 13{\cdot}6 = 6836$ grammes.]

8. A stone, whose volume is a cubic decimetre, and whose specific gravity is 2·5, is wholly immersed in a vessel of water, and supported by a string without touching the bottom or the sides; required first, to find the tension of the string; secondly, to say whether or no there is any increase of pressure on the bottom of the vessel, and if there is, how you would proceed to determine its amount.

[The tension of the string is equal to the weight of the stone, 2·5 kilos, minus the upward pressure of the water, 1 kilo; it is, therefore, 1·5 kilos. The increase of pressure on the bottom of the vessel depends on the increase of height in the level of the water, which of course depends on the shape of the vessel. If A represent the area of the base, y the height of the water before the stone is immersed, and x the increase of height, then the total pressure on the base is equal to the weight of a volume of water $A(y + x)$, and the pressure, before the immersion of the stone, was equal to the weight of a volume Ay; the increase, therefore, is equal to the weight of a volume Ax.]

9. A platinum sphere, 3 centimetres in diameter, is suspended from the scale of a balance, and wholly immersed in mercury. It is exactly counterbalanced by a cylinder of copper of the same diameter, wholly immersed in water. Find the height of the cylinder, the specific gravity of platinum being 21·5, of mercury 13·6, and of copper 8·8. *Ans.* 2·0256 centimetres.

[From the conditions of the problem the weight of the sphere, minus the weight of the mercury displaced, is equal to the weight of the cylinder, minus the weight of the water displaced. That is, if we take r for the radius of the sphere and of the cylinder, and h for the height of the cylinder,

$$\frac{4}{3}\pi r^3 \times 21{\cdot}5 - \frac{4}{3}\pi r^3 \times 13{\cdot}6 = \pi r^2 \times h \times 8{\cdot}8 - \pi r^2 \times h$$

$$\frac{4}{3}\pi r^3(21{\cdot}5 - 13{\cdot}6) = \pi r^2 h(8{\cdot}8 - 1) \text{ or } \frac{4}{3}\pi r^3 \times (7{\cdot}9) = \pi r^2 h \times (7{\cdot}8)$$

$$\therefore \frac{4}{3} r\,(7{\cdot}9) = h\,(7{\cdot}8) \text{ and } h = \frac{\frac{4}{3} \times 1{\cdot}5 \times 7{\cdot}9}{7{\cdot}8} = 2{\cdot}0256.]$$

10. The specific gravity of a sample of milk, diluted with water, is found to be 1·024, while the specific gravity of pure milk is 1·03; find what proportion of water has been added to the milk. *Ans.* 1 volume of water to 4 of milk.

[Let x be the volume of milk, and y the volume of water. Then $x \times 1·03 + y \times 1 = (x + y) \times 1·024$, from which we get $6x = 24y$ and $x = 4y$.]

11. Find the weight, in ounces, of a silver globe, of which the radius is 3 inches, the specific gravity of silver being 10·5. *Ans.* 687·5 oz.

12. A body weighing 450 grains in air, loses 210 grains in water, and 150 grains in spirit; find the specific gravity of the body, and of the spirit. *Ans.* (1) 2·143; (2) ·714.

[(1) $\frac{450}{210} = 2·143$; (2) $\frac{150}{210} = ·714$ nearly.]

13. A globe, 2 feet in diameter, when floating in water, is half immersed; find its weight. *Ans.* 130·9 lbs.

[The volume of the water displaced is half the volume of the globe, or $\frac{2}{3}\pi r^3$, that is, $\frac{2}{3} \times 3·1416 \times 1 = 2·0944$ cubic feet. The weight of this volume of water is $2·0944 \times 62·5 = 130·9$ lbs., which must, of course, be equal to the weight of the globe.]

14. If the barometer rises from 750 to 760 millimetres, what is the increase of pressure on a square metre? *Ans.* 136 kilogrammes.

[It is equal to the weight of a column of mercury having a base of one square metre and a height of one centimetre. A square metre is equal to 100 square decimetres, and a centimetre is equal to ·1 of a decimetre; therefore the column of mercury will be $100 \times ·1 = 10$ cubic decimetres in volume; and its weight will be $10 \times 13·6 = 136$ kilogrammes.]

15. Find the pressure on one square inch, in lbs., when the barometer stands at 29·5 inches. *Ans.* 14·5 nearly.

16. Mercury is poured into a very long barometer tube until only one inch, in length, of the tube is occupied with air, which exists at the pressure of the atmosphere; the tube being then inverted and plunged into a cistern of mercury, the inclosed air expands and occupies 15 inches of the length of the tube, while a column of mercury, 28 inches high, is supported below it. Find the true barometric height. *Ans.* 30 inches.

[Let V represent the volume of the inclosed air before inversion, and P its pressure, V′ and P′ its volume and pressure after inversion. Then, from Boyle's law, $P' = \frac{VP}{V'} = \frac{1}{15}P$. Now, from the conditions of the problem, P is equal to the pressure of the atmosphere, and, after inversion, the pressure of the atmosphere balances P′ plus 28 inches of mercury; that is, $P = 28 + P' = 28 + \frac{P}{15}$, and $P - \frac{P}{15} = 28$ $\therefore$ $14P = 28 \times 15$ and $P = \frac{28 \times 15}{14} = 30$.]

17. Find the greatest height to which a common pump can raise a liquid, whose specific gravity is 1·7, when the barometer stands at 29 inches, the specific gravity of mercury being 13·6. *Ans.* 19⅓ feet.

18. The diameter of the piston of a lifting pump is 1 foot, the range of the piston is 2·5 feet, and it makes 8 strokes a minute; find the weight of water discharged, per minute, supposing that the highest level of the piston range is not higher than the height to which water will rise under the pressure of the atmosphere. *Ans.* 981·75 lbs.

[The volume of the water discharged, per minute, is $\pi r^2 \times 2{\cdot}5 \times 8 = 3{\cdot}1416 \times {\cdot}25 \times 8 = 15{\cdot}708$ cubic feet or $15{\cdot}708 \times 62{\cdot}5 = 981{\cdot}75$ lbs.]

19. The area of the piston of a forcing pump is 8 square inches; find what force must be exerted to push it down when the water in the water in the ascending tube is 30 feet above it. *Ans.* 104·16 lbs.

20. In a condenser the space not traversed by the piston is one-fiftieth of the internal volume of the condenser; find the limit of the condensing power, in atmospheres. *Ans.* 50.

[The limit is reached when the tension (T) of the condensed air is equal to the tension of the volume (v) of air in the condenser when compressed into the untraversed space (v'); that is, when $T = H\frac{v}{v'}$, or, in the case before us, when $T = 50H$.]

21. What must be the ratio of the space not traversed by the piston to the internal volume of the condenser, in order that the condenser shall be just able to produce a pressure of 40 atmospheres? *Ans.* 1 : 40.

[In the case proposed, a pressure of 40 atmospheres represents the limit of the condensing power; therefore the formula $T = H\frac{v}{v'}$ becomes $40\,H = H\frac{v}{v'}$, and $\frac{v}{v'} = 40$; in other words, the ratio of the untraversed space to the volume of the condenser is 1 : 40.]

SECOND YEAR'S COURSE:

SOUND AND HEAT.

MONTHLY EXAMINATION PAPER, No. 1.

SOUND, SECTIONS I., II.,

A. Questions.

1. Explain the nature of wave motion, as distinct from motion of translation, and apply your explanation to the waves of sound.

2. Explain as fully as you can what is meant by the Length, the Amplitude, and the Form of a wave of sound.

3. State in what way the velocity of sound in air depends on the elastic force, the density, and the temperature of the air in which it is transmitted; and explain why it is that, if the temperature of the air continues constant, while the density and elastic force vary, the velocity of sound remains unchanged.

[The greater the elastic force of the medium, the greater the velocity of sound; the greater the density, the less the velocity of sound. Strictly expressed, the law is that the velocity of sound varies directly as the square root of the elastic force, and inversely as the square root of the density, or $V \propto \frac{\sqrt{E}}{\sqrt{D}}$. Now when the temperatur of the air increases, without any change of barometric pressure, the density is diminished while the elastic force remains the same; therefore $\frac{\sqrt{E}}{\sqrt{D}}$ is increased, and the velocity of sound is increased. But so long as the temperature remains the same, the density, according to Boyle's law, varies as the pressure, which is always equal to the elastic force; that is $D \propto E$, and the ratio $\frac{E}{D}$ and $\frac{\sqrt{E}}{\sqrt{D}}$ remain unchanged; therefore the velocity of sound, which varies as $\frac{\sqrt{E}}{\sqrt{D}}$ remains the same.]

4. Describe the construction of the siren, and explain how the rate of vibration of any note may be determined by means of it.

5. What is meant by the interference of sound waves? Explain how the blending of two sounds may produce silence; and describe one or two experiments by which this phenomenon may be illustrated.

6. Describe the Resonators of Helmholtz, and explain how they may be employed for the analysis of musical sounds.

7. Prove that the intensity of sound varies inversely as the square of the distance from the sounding body; and show that this principle does not apply to the case of speaking-tubes.

8. If, when two trains are passing one another, the whistle of one happens to be sounding, a distinct fall in the pitch of the note is perceived by a passenger in the other train, at the moment the whistle passes him. Explain fully the physical cause of this phenomenon.

9. If two strings, stretched on the same sounding board, are tuned to exactly the same pitch, and one of them is put in vibration, the other will vibrate too, although it has not been touched; but if the strings are slightly out of tune with one another, this effect will not be produced. Give an account of the physical process that goes on in each case.

10. Explain the physical cause of Beats in music, and describe some experiment by which the phenomenon may be strikingly illustrated.

B. Exercises.

1. An interval of 3·75 seconds is observed to elapse between a flash of lightning and the corresponding peal of thunder, find the distance of the lightning, supposing the thermometer at the time to mark 15° C. *Ans.* 4,200 feet.

[The velocity of sound in air at 15° C. is 1,120 feet a second; therefore the distance travelled in 3·75 seconds is $1{,}120 \times 3{\cdot}75 = 4{,}200$ feet.]

2. The flash of a gun is seen 5 seconds before the report is heard, the temperature being 5° C.; find the distance of the gun. *Ans.* 5,500 feet.

[The velocity of sound in air increases about 2 feet a second for every degree Centigrade; therefore, as the velocity is 1,120 at 15° C., it is 1,100 at 5° C.; hence the distance is $1{,}100 \times 5 = 5{,}500$ feet.]

3. The density of Oxygen, compared to that of Hydrogen, at the same temperature and under the same pressure, is as 16 to 1; what would be the velocity of sound in Oxygen as compared with its velocity in Hydrogen, under the same circumstances? *Ans.* 1 : 4.

[If V represent the velocity of sound, E the elastic force of the medium, and D its density, $V \propto \frac{\sqrt{E}}{\sqrt{D}}$, and when E is constant, $V \propto \frac{1}{\sqrt{D}}$. In the case before us E is constant, because the two gases are supposed to be under the same pressure, and the elastic force of each exactly balances the pressure; therefore the velocity of sound in the Oxygen is to the velocity of sound in the Hydrogen as $\frac{1}{\sqrt{16}} : \frac{1}{\sqrt{1}}$, that is, as 1 : 4.]

4. The velocity of sound in Hydrogen, at 0° C., is 4,164 feet a second; find the length of a sound wave produced in Hydrogen, at 0° C., by a note that vibrates 512 times a second. *Ans.* 8·13 feet.

[If V represent the velocity of sound, n the number of vibrations in a second, and L the length of a sound wave, then $L = \frac{V}{n}$ which, in the present case, is $\frac{4,164}{512} = 8{\cdot}13$.]

5. The amplitude of the vibrations of a tuning fork is a quarter of an inch, and the distance of the listener is 6 feet; what change would take place in the intensity of the sound heard, if the amplitude of the vibrations were increased to half an inch, and the distance of the listener to 4 feet? *Ans.* It would be increased 9 fold.

[The intensity of a sound is directly proportional to the square of the amplitude of vibration, and inversely proportional to the square of the distance; that is $I \propto \frac{A^2}{R^2}$, I being the intensity of the sound, A the amplitude of vibration, and R the distance. Hence, in the case before us, the intensity of the sound heard, in the one case, is to the intensity of the sound heard in the other, as $\frac{(\frac{1}{2})^2}{16} : \frac{(\frac{1}{4})^2}{36}$, that is, as 9 : 1.]

6. What is the period of vibration of a whistle which gives waves 3 feet long, in air, at a temperature of 10° C? *Ans.* $\frac{1}{370}$ of a second.

[$n = \frac{V}{L} = \frac{1110}{3} = 370$; that is, the number of vibrations per second is 370, and the period of one vibration is, therefore, $\frac{1}{370}$ of a second.]

7. Find the length of the sound waves produced by the note Sol, when the fundamental note Do vibrates 256 times in a second, and the temperature of the air is 15° C. *Ans.* 2 feet 11 inches.

[The rate of vibration of Sol, as compared with Do, is as 3 : 2; therefore, in the case before us, the number of vibrations per second is $256 \times \frac{3}{2} = 384$. Taking then the formula, $L = \frac{V}{n}$, we have $L = \frac{1120}{384} = 2$ feet 11 inches.]

8. The pitch of the ordinary speaking voice of a man corresponds to about the middle of the range of a barytone, and extends over an interval of a fourth, say from F in the bass clef to the B flat above it

(see Hullah, The Cultivation of the Speaking Voice, p. 25); find the corresponding range in the length of the sound waves he produces, when speaking. *Ans.* From 6·56 to 4·92 feet.

[Taking the rate of vibration of the middle C of the piano at 256, we shall get that of the F, a fifth below, $256 \times \frac{2}{3} = 170\cdot\dot{6}$, and of B flat, which may be regarded a fourth above F, $170\cdot 6 \times \frac{4}{3} = 227\cdot\dot{6}$. Hence for F, $L = \frac{1120}{170\cdot 6} = 6\cdot 56$, and for B flat, $L = \frac{1120}{227\cdot 6} = 4\cdot 92$.]

9. In Professor Tyndall's book on Sound it is stated, that "the waves generated by a man's voice, in common conversation, are from 8 to 12 feet in length"; what is the range of pitch corresponding to this range in the length of the sound waves? *Ans.* From G on the first line of the bass clef to D on the third line.

[$L = \frac{V}{n}$ $\therefore$ $n = \frac{V}{L}$; consequently, when $L = 12$ feet, at a temperature of 15° C., $n = \frac{1120}{12} = 93\cdot\dot{3}$; and when $L = 8$, $n = \frac{1120}{8} = 140$. Now since the vibrations of C in the bass clef are 128 in a second, those of G, the fourth below, will be $128 \times \frac{3}{4} = 96$; and those of D, immediately above it, will be $128 \times \frac{9}{8} = 144$. The range, therefore, of a man's voice, in conversation, according to the statement of Professor Tyndall, is from about G, on the first line of the bass clef, to D on the third line. The estimate given by Professor Hullah, and referred to in the last exercise, is much higher, and I think nearer to the truth.]

10. By what interval does the first third of the major scale exceed the first third of the minor scale? *Ans.* $\frac{25}{24}$.

[The first third of the major scale is C E, and the interval is the ratio 5 : 4, or $\frac{5}{4}$; the first third of the minor scale is E G, and the interval is the ratio $\frac{3}{2} : \frac{5}{4}$, which is 6 : 5, or $\frac{6}{5}$. Hence the interval by which the former exceeds the latter is the ratio $\frac{5}{4} : \frac{6}{5}$, which is 25 : 24, or $\frac{25}{24}$.]

11. It has been shown, by means of the siren, that the wings of a gnat vibrate 15,000 times a second; what will be the length of the corresponding sound waves, the temperature being 15° C.? *Ans.* ·89 of an inch.

$$[L = \frac{V}{n} = \frac{1120 \times 12}{15{,}000} = \frac{13440}{15000} = \cdot 89 \text{ of an inch.}]$$

12. A railway train passes through a station, at the rate of 45 miles an hour; as it passes, the whistle is sounding, and produces a note which corresponds to 560 vibrations a second. Find the pitch of the note heard by a person standing close to the edge of the platform, first as the train approaches, and secondly, as the train recedes. *Ans.* (1) 593 vibrations a second; (2) 527 vibrations a second.

[The note heard, as the train approaches, corresponds to $n + n'$ vibrations in the second, and as the train recedes, to $n - n'$ vibrations in the second; n being the number of vibrations actually emitted by the whistle in a second, and n' the number of sound waves that occupy the space traversed by the train in a second. In the case before us $n = 560$ $\therefore$ the wave length $= \frac{1120}{560} = 2$ feet; and the train traverses 66 feet in a second. Therefore $n' = \frac{66}{2}, = 33$; and $n + n' = 560 + 33 = 593$, while $n - n' = 560 - 33 = 527$.]

13. Under the conditions given above, what is the interval through which the note will fall, as heard by the observer, at the moment the whistle passes? *Ans.* A major tone nearly.

[The interval between the two notes is $\frac{593}{527}$, which is almost exactly equal to $\frac{9}{8}$, a major tone.]

14. Prove that, in such cases as that considered in the last two exercises, the interval through which the note falls is independent of the pitch, and depends only on the velocity of sound and the velocity of the train.

[If we adopt the symbols above employed, the interval will always be $\frac{n + n'}{n - n'}$. Now if v be the velocity of the train, in feet per second, and L the wave length, in feet, it is evident that $n' = \frac{v}{L}$; but $L = \frac{V}{n}$, V being the velocity of sound, in feet per second. Therefore $n' = v \div \frac{V}{n} = \frac{nv}{V}$; and $n + n' = n + \frac{nv}{V} = n\left(1 + \frac{v}{V}\right)$. In like manner, $n - n' = n - \frac{nv}{V} = n\left(1 - \frac{v}{V}\right)$. Hence the interval $\frac{n + n'}{n - n'}, = \frac{n\left(1 + \frac{v}{V}\right)}{n\left(1 - \frac{v}{V}\right)} = \frac{\frac{V + v}{V}}{\frac{V - v}{V}} = \frac{V + v}{V - v}$; and it is, therefore, independent of n, the pitch of the note, and depends only on V and v, the velocity of sound and the velocity of the train. The same result may be obtained perhaps more simply thus: $n = \frac{V}{L}, n' = \frac{v}{L}$ $\therefore$ $\frac{n + n'}{n - n'} = \frac{\frac{V}{L} + \frac{v}{L}}{\frac{V}{L} - \frac{v}{L}} = \frac{V + v}{V - v}$.]

15. At what rate must two trains pass one another, in order that, if the whistle of one is sounding, an observer in the other shall perceive a fall of pitch in the note heard, at the moment of passing, equal to a major third, the temperature of the air at the time of observation being 10° C.? *Ans.* Their relative motion must be 123⅓ feet per second, or about 85 miles an hour.

[Let us first suppose the train, in which the observer is stationed, to be at rest, and the other, in which the whistle is sounded, to pass it by. The fall of pitch that will be noticed, as the train passes, will be, from the last exercise, $\frac{V+v}{V-v}$; and when this interval is a major third, $\frac{V+v}{V-v}=\frac{5}{4}$, or $4V+4v=5V-5v$; and $V=9v$. Hence, in order to produce a fall of a major third, v, the velocity of the moving train, must be equal to $\frac{V}{9}=\frac{1110}{9}=123\frac{1}{3}$ feet per second, or about 85 miles an hour. Now, if the two trains are moving in opposite directions, it is clear that the same effect will be produced whenever the sum of their velocities is equal to 85 miles an hour.]

MONTHLY EXAMINATION PAPER, No. 2.

SOUND, SECTIONS, III., IV., V.

A. Questions.

1. State the position of the node and ventral segments of an open organ pipe, when the fundamental tone is sounded; also when the first overtone is sounded. Describe an experiment by which your answer may be illustrated.

2. Explain the physical process by which the phenomenon of resonance is produced when a column of air is thrown into vibration by a musical note, and show the relation that must exist between the length of the column of air and the pitch of the note, when the resonance is most perfect. Describe also some experiments to illustrate your answer.

3. State exactly how the pitch of a musical note, produced by the transverse vibrations of a stretched wire, depends on (1) its length, (2) its tension, (3) its thickness, and (4) its density; and give some examples to illustrate your answer.

[If n be the number of vibrations in a second, L the length of the wire, T its tension, R the radius of its cross section, D its density, then (1) $n \propto \frac{1}{L}$, (2) $n \propto \sqrt{T}$, (3) $n \propto \frac{1}{R}$, (4) $n \propto \frac{1}{\sqrt{D}}$.]

4. Explain what is meant by the harmonics, or overtones, of a vibrating string, and describe some experiments by which their existence may be demonstrated.

5. While an open organ pipe is sounding I cover the open end with my hand; state what change will take place in the pitch of the note, and explain the physical cause of the change.

6. It was found, long ago, by piano-forte makers, that the tone of a note is most pleasing to the ear when the wire is struck by the hammer at a distance, from one end, equal to about one-seventh of its whole length; give a scientific reason why this should be the case.

7. I take a square brass plate, fixed, at its central point, to an upright stand, and scatter fine sand lightly over its surface. I then put the plate in vibration by means of a strong violin bow; a clear distinct note is heard, and the sand, in a moment, arranges itself in a definite pattern on the plate. Again I bow the plate, producing a different note from before, and the sand makes a new pattern different from the first. Explain the physical cause of this phenomenon, showing why the sand sometimes arranges itself in one way, and sometimes in another.

8. Explain the theory of Helmholtz regarding the nature of the vowel sounds; and show how this theory may be supported experimentally, (1) by resolving the vowel sounds into their component elements, and (2) by compounding vowel sounds from simple musical tones.

9. Taking the range of one octave in the diatonic scale, state what combinations of two notes are harmonious, and what combinations are discordant; write down the several ratios of the notes that produce the harmonious combinations, and of the notes that produce the discordant combinations; and explain, as far as you can, why the one set of combinations produce an agreeable, the other set of combinations a disagreeable effect on the ear.

10. Explain, as far as you can, in what way the Loudness, the Pitch, and the Quality of musical sounds are actually represented within the inner chamber of the ear; and by what means the sense of hearing is able to discriminate between the different varieties of each.

B. Exercises.

1. A stretched wire, when put in vibration, executes 200 vibrations a second; find the number of vibrations, per second, that will be executed respectively by $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{1}{2}$ of the same wire; and name the

corresponding notes of the musical scale, calling the note produced by the whole string Do. *Ans.* (1) 250, 300, 400; (2) Mi, Sol, Do.

[The rate of vibration varies inversely as the length of the string, all other conditions remaining the same. Therefore if n be the number of vibrations, per second, executed by the whole string, $\frac{4}{5}$ of the string will give $\frac{5}{4}n$, $\frac{2}{3}$ of the string will give $\frac{3}{2}n$, $\frac{1}{2}$ of the string will give $2n$. Further, the vibrations of the successive fractions of the string, compared with the vibrations of the whole string, are in the ratios respectively, of 5 : 4, a major third, 3 : 2, a fifth, and 2 : 1, an octave; therefore the series of notes, beginning with that of the whole string, will be Do, Mi, Sol, Do.]

2. A wire, stretched by a weight of 4 kilogrammes, executes 200 vibrations a second; find the number of vibrations, per second, it will execute if, without any change in length, it is stretched successively by weights of 9, 16, and 25 kilogrammes; and give the names of the series of notes produced, calling the lowest Do. *Ans.* (1) 300, 400, 500; (2) Do, Sol, Do, Mi.

[The rate of vibration is proportional to the square root of the tension, or stretching weight; therefore, in the present case, it will be, for the whole series, as $\sqrt{4} : \sqrt{9} : \sqrt{16} : \sqrt{25}$, or as $2 : 3 : 4 : 5$; and the notes will be evidently, in ascending series, Do, Sol, Do, Mi.]

3. Two stretched wires, one of platinum, specific gravity 22, the other of iron, specific gravity 7·8, of equal diameters, and stretched by equal weights, are found, when put in vibration, to yield exactly the same musical note; find their respective lengths. (Baccalauréat, Paris.) *Ans.* ·5954 : 1.

[In this case, since the tension of the two wires is the same and also the thickness, the rate of vibration is inversely proportional to the product of the length into the square root of the density. Hence if n, L, D, and n', L$'$, D$'$, represent the number of vibrations per second, the length, and the density, of the platinum and iron wires respectively, we have $n : n' :: \mathrm{L}'\sqrt{\mathrm{D}'} : \mathrm{L}\sqrt{\mathrm{D}}$; but, from the conditions of the problem, $n = n'$, therefore $\mathrm{L}'\sqrt{\mathrm{D}'} = \mathrm{L}\sqrt{\mathrm{D}}$, and $\mathrm{L} = \mathrm{L}'\frac{\sqrt{\mathrm{D}'}}{\sqrt{\mathrm{D}}} = \mathrm{L}'\sqrt{\frac{7\cdot8}{22}} = \mathrm{L}' \times \sqrt{\cdot354} = \mathrm{L}' \times \cdot5954$.]

4. Find the length of a tube, closed at one end, that will resound most perfectly to the note of a tuning fork vibrating 512 times in a second, it being supposed that the tube is long in proportion to its diameter, and that the temperature is 15° C. *Ans.* 6·5625 inches.

[It is one-fourth the length of the sound wave, that is, L; but $\mathrm{L} = \frac{\mathrm{V}}{n} = \frac{1120}{512} = 2\cdot1875$ feet; therefore the length of the tube is $\frac{2\cdot1875}{4} = \cdot546875$ of a foot, or $\cdot546875 \times 12 = 6\cdot5625$ inches.]

5. A wire, stretched over a sounding board, when put in vibration, executes 128 vibrations a second. I shorten the wire, by means of a moveable bridge, first to one half, then, successively, to one third, one fourth, one fifth, and one sixth of its whole length, each time putting the string in vibration, and each time getting a higher note than before. State what is the rate of vibration, in each case; and further, calling the first note Do, name the successive notes produced.

[The rate of vibration is inversely proportional to the length of the string; therefore, since the successive lengths are as $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$, the successive rates of vibration will be as $1 : 2 : 3 : 4 : 5 : 6$; and the number of vibrations, per second, will be respectively, 128, $128 \times 2 = 256$, $128 \times 3 = 384$, $128 \times 4 = 512$, $128 \times 5 = 640$, $128 \times 6 = 768$. From an inspection of the rates of vibration it will be evident to the student that the successive notes are, (1) Do, (2) Do, the octave above, (3) Sol, the fifth above the octave, (4) Do, the second octave, (5) Mi, the third above the second octave, and (6) Sol, the fifth above the second octave.]

MONTHLY EXAMINATION PAPER, No. 3.

HEAT, Sections I., II.

A. Questions.

1. Describe the process of making a thermometer.

2. Explain the graduation of the Centigrade, the Fahrenheit, and the Réaumur scales; and deduce the formulas for converting the reading of any one scale into the corresponding reading of any other.

3. Explain what is meant by the maximum density of water; and give the temperature at which water reaches its maximum density, in terms of the Fahrenheit and of the Centigrade scales.

4. Explain accurately and fully the meaning of the term *coefficient of expansion;* and apply your explanation to the three kinds of expansion, linear, superficial, and cubical.

5. Prove that, if k represent the coefficient of linear expansion, $2k$ will be the coefficient of superficial expansion, and $3k$ the coefficient of cubical expansion, approximately.

6. Give the coefficient of linear expansion for some of the more familiar metals between 0° C. and 100° C.

7. A compound bar, composed of a plate of copper and a plate of zinc, rivetted together, is flat at 20° C.; what will be its form at 4° C., and at 36° C.?

8. Explain the principle of the metallic thermometer, and describe the construction of some form of it.

9. Describe some of the practical results and practical applications of the expansion of solids by heat.

10. Explain the principle and the construction of what is called the gridiron compensation pendulum.

11. Why is it that platinum wire may be fused into a glass tube, without danger of its cracking the glass in cooling, whereas silver wire can not?

B. Exercises.

1. Convert the following readings of the Fahrenheit scale into the corresponding readings Centigrade; (1) 100° F., (2) 45·5° F., (3) 19° F., (4) — 7 F. *Ans.* (1) 37·$\dot{7}$° C., (2) 7·5° C., (3) — 7·$\dot{2}$° C., (4) — 21·$\dot{6}$° C.

[If C represent the Centigrade reading, and F the Fahrenheit reading, $C = (F - 32) \times \frac{5}{9}$.]

2. Convert the following readings of the Centigrade scale into the corresponding readings Fahrenheit: (1) 75° C., (2) 17° C., (3) — 39° C. *Ans.* (1) 167° F., (2) 62·6° F., (3) — 38·2° F.

[If F represent the Fahrenheit reading, and C the Centigrade, $F = \frac{9}{5} C + 32$.]

3. Convert the following readings of the Réaumur scale into the corresponding readings Fahrenheit and Centigrade respectively: (1) 5° R., (2) 20° R., (3) — 18° R. *Ans.* (1) 43·25° F., 6·25° C., (2) 77° F., 25° C., (3) — 7·5° F., — 22·5° C.

[If R, F, and C, represent the readings on the three scales respectively, $F = \frac{9}{4} R + 32$, $R = (F - 32) \times \frac{4}{9}$, $C = \frac{5}{4} R$, $R = \frac{4}{5} C$.]

4. Find at what temperature the readings of the Fahrenheit and Centigrade scales will exactly correspond. *Ans.* — 40°.

[Since $C = (F - 32) \frac{5}{9}$, it follows that, when $C = F$, $(F - 32) \frac{5}{9} = F$, or $\frac{5}{9} F - \frac{5}{9} 32 = F \therefore 5 F - 160 = 9 F$ and $F = -40$.]

5. The length of an iron rail, at 0° C., is 15 feet; find its length at 30° C., the coefficient of expansion being ·0000122. *Ans.* 15·00549.

[Formula for linear expansion; if l_0 be the length of a body at 0° C., l_t its length at $t°$, and k the coefficient of expansion, $l_t = l_0(1 + kt)$, and $l_0 = \frac{l_t}{1 + kt}$. In the present case, $l_t = 15\,(1 + \cdot0000122 \times 30) = 15{\cdot}00549$.]

6. If a line of railway is laid on a day that the temperature is 0° C., the rails being 18 feet long and the coefficient of expansion ·0000122, what distance must they be placed apart in order that they shall be just touching when the temperature has risen to 50° C.? *Ans.* ·132 of an inch.

[The expansion of each rail will be lkt, l being length at 0° C., k the coefficient of expansion, and t the increase of temperature in degrees Centigrade; that is, $18 \times \cdot0000122 \times 50 = \cdot01098$ of a foot, or ·132 of an inch nearly.]

7. The hot water pipes, running along a line of building, are 100 feet in length when they are set in their place, the temperature being, at the time, 10° C.; what space must be allowed for their expansion when the temperature rises to 100° C., the coefficient of expansion of the metal being ·0000112? *Ans.* ·1 of a foot.

[If l_0 be the length of a body at 0° C., l_t its length at $t°$, l_m its length at any other temperature $m°$, and k its coefficient of expansion, then $l_t = l_0(1 + kt)$, $l_m = l_0(1 + km)$, and $l_0 = \frac{l_m}{1 + km}$. Comparing the first and third of these formulas we get $l_t = \frac{l_m}{1 + km} \times (1 + kt)$. This will give, in the case before us, for the length of the pipes at 100° C.,

$$l_t = \frac{100}{1 + \cdot0000112 \times 10} \times 1 + \cdot0000112 \times 100 = 100{\cdot}1 \text{ feet.}]$$

8. A bar of metal which, at 0° C., is 1·28 metres long, is placed in a furnace where it is found to increase in length to 1·2915 metres; find the temperature of the furnace, the mean coefficient of expansion of the metal, between 0° C. and 600° C., being ·000017. *Ans.* 528·5° C.

$$[l_t = l_0(1 + kt) = l_0 + l_0 kt;\ \therefore\ t = \frac{l_t - l_0}{l_0 k} = \frac{1{\cdot}2915 - 1{\cdot}28}{1{\cdot}28 \times \cdot000017} = 528{\cdot}5.]$$

9. The metre is defined to be the distance between two points on a certain platinum bar, called the standard metre, when the temperature of the bar is 0° C.; what is the distance between these points when the temperature of the bar is 16° C., the coefficient of expansion of platinum being ·0000088? *Ans.* 1·0001408 metres.

$$[l_t = l_0(1 + kt) = 1 \times (1 + \cdot0000088 \times 16) = 1{\cdot}0001408.]$$

10. Given the coefficient of linear expansion for forged iron to be ·0000122, and for brass ·00001875, for one degree Centigrade, find

the corresponding coefficients for one degree Fahrenheit. *Ans.* (1) $.000006\dot{7}$; (2) $\cdot00001041\dot{6}$.

[A degree Fahrenheit is $\frac{5}{9}$ of a degree Centigrade; therefore the expansion for one degree Fahrenheit will be $\frac{5}{9}$ of the expansion for one degree Centigrade; that is, $\cdot0000122 + \frac{5}{9} = \cdot000006\dot{7}$, and $\cdot00001875 \times \frac{5}{9} = \cdot00001041\dot{6}$.]

11. A bar of platinum is 3 metres long at 10° C.; find its length at 100° C., the coefficient of expansion being ·0000088 (Baccalauréat, Paris.) *Ans.* 3·00237 metres.

[Taking l_o, l_t, l_m, to represent the length of the bar at 0°, t°, m°, respectively, and k for the coefficient of linear expansion, $l_t = \frac{l_m}{1+km} \times (1+kt)$; hence, in the present case, $l_t = \frac{3}{1+\cdot0000088\times10} \times (1 + \cdot0000088 \times 100) \doteq 3\cdot00237$.]

12. A sheet of lead has an area of 48 square feet at 0° C., what will its area be at 40° C., the coefficient of linear expansion of lead being ·00002855? *Ans.* 48·11 square feet.

[The coefficient of superficial expansion is twice the coefficient of linear expansion, that is, in the present case, $\cdot00002855 \times 2 = \cdot0000571$. Taking k to represent this fraction, A_0 the area at 0° C., and A_t the area at any other temperature t°, $A_t = A_0(1 + kt) = 48 \times (1\cdot0000571 \times 40) = 48\cdot11$.]

13. A sheet of copper has an area of 25 square inches at 0° C.; what will be its area at 38° C., the coefficient of linear expansion of copper being ·00001715? *Ans.* 25·032585 square inches.

[$A_t = A_0(1 + kt) = 25\,(1 + \cdot0000343 \times 38) = 25\cdot032585$.]

14. The volume of a mass of cast iron is 245 cubic feet, at 0° C.; find its volume at 100° C., the coefficient of linear expansion of cast iron being ·00001125. *Ans.* 245·826875 cubic feet.

[The coefficient of cubical expansion is three times the coefficient of linear expansion; therefore, in the present case, it is ·00003375. Taking k to represent this fraction, V_0 the volume at 0° C., and V_t the volume at t°, we have $V_t = V_0(1 + kt)$, which in the present case will be, $245\,(1 + \cdot00003375 \times 100) = 245\cdot826875$.]

15. The volume of a mass of platinum, at 0° C., is 28·5 cubic inches; find its volume at 56° C., taking the coefficient of linear expansion of platinum to be ·00000875. *Ans.* 28·541895 cubic inches.

[$V_t = V_0(1 + kt) = 28\cdot5\,(1 + \cdot00002625 \times 56) = 28\cdot541895$.]

16. The specific gravity of silver, at 0° C., is 10·5.; find its

specific gravity at 100, the coefficient of linear expansion being ·00001905. *Ans.* 10·44.

[The specific gravity of a body is inversely proportional to its volume when the mass is constant. Therefore, if V_0, V_t, and S_0, S_t represent the volumes and specific gravities of a mass of silver at 0° C., and at any other temperature $t°$ C., respectively, $\frac{S_t}{S_0} = \frac{V_0}{V_t}$, and $S_t = \frac{V_0 S_0}{V_t}$. But from the formulas for expansion we have $V_t = V_0 (1 + kt)$. Therefore $S_t = \frac{V_0 S_0}{V_0 (1 + kt)} = \frac{S_0}{1 + kt}$. In the case before us, $k = \cdot00001905 \times 3 = \cdot00005715$, $\therefore S_t = \frac{10\cdot5}{1\cdot005715} = 10\cdot44$.]

MONTHLY EXAMINATION PAPER, No. 4.

HEAT, SECTIONS III., IV.

A. Questions.

1. State precisely in what respect water is an exception to the general law that bodies expand with heat and contract with cold; and point out some of the consequences of this fact in the phenomena of Nature.

2. Describe an experiment to prove that water attains its maximum density at about 4° C.

3. Explain the difference between the apparent and the real expansion of a liquid.

4. Describe some process by which the coefficient of real expansion of mercury may be determined.

5. What is meant by the reduction of the Barometer reading to 0° C.? What is the object of this reduction? and how is it made?

6. Explain what is meant by the coefficient of expansion of a gas under constant pressure; and state what it is.

7. Describe the Air Thermometer, and point out some of its advantages.

8. What is meant by the absolute zero of temperature? State what it is, in degrees Fahrenheit and Centigrade.

9. What is the reason that a gas is chilled as it expands? Explain as far as you can the connection between this fact and the dynamical theory of heat.

10. Explain how the volume of a given quantity of a gas depends on pressure and temperature; and deduce the formula by which, if the

volume, under any pressure, and at any temperature, is given, you can find the volume at 760 millimetres of pressure and 0° C.

B. **Exercises.**

1. What is the volume at 100° C. of a certain quantity of mercury which at 0° C. has a volume of 100 cubic inches? The coefficient of expansion of mercury may be taken as ·00018. *Ans.* 100·8 inches.

$$[V_t = V_0(1 + kt) = 100 \times 1{\cdot}018 = 101{\cdot}8.]$$

2. The reading of the Barometer at 15° C. is 763 millimetres; what would the reading be, under the same pressure, at 0° C.? *Ans.* 760·945 mm.

[If h_0, h_t represent the Barometric height at 0° C., and at any other temperature t° C., and k the coefficient of expansion of mercury, $h_0 = \frac{h_t}{1 + kt}$; which will give, in the present case, $h_0 = \frac{763}{1 + {\cdot}00018 \times 15} = 760{\cdot}945$.]

3. If the reading is 29·5 inches at 10° C., what is the corrected reading for 0° C.? *Ans.* 29·447 inches.

4. What is the volume at 20° C. of 100 cubic inches of air measured at 0° C., the pressure remaining the same? The coefficient of expansion under constant pressure is ·00366. *Ans.* 107·32 inches.

$$[V_t = V_0\,(1 + kt) = 100 \times 1{\cdot}0732 = 107{\cdot}32.]$$

5. What is the volume, in cubic inches, at 0° C., of one cubic foot of air measured at 100° C., the pressure remaining constant? *Ans.* 1264·896 inches.

$$[V_0 = \frac{V_t}{1+kt} = \frac{100}{1{\cdot}366} = {\cdot}732 \text{ cubic foot or } {\cdot}732 \times 1728 \text{ cubic inches.}]$$

6. The Barometer reading is 750 millimetres at 16° C.; what will it be when reduced to 0° C.? *Ans.* 747·846 inches.

$$[h_0 = \frac{h_t}{1+kt} = \frac{750}{1{\cdot}00288} = 747{\cdot}846.]$$

7. A mass of air, which measures a hundred cubic centimetres at 0° C., is found to measure, at other temperatures, 96, 106, and 120 cubic centimetres respectively, the pressure remaining constant; find the temperatures. *Ans.* (1) —13·66° C. (2) 16·393° C. (3) 54·645° C.

[$V_t = V_0(1 + kt) = V_0 + V_0 kt \therefore t = \frac{V_t - V_0}{V_0 k}$. Applying this formula we get (1) $t = \frac{-5}{{\cdot}366} = -13{\cdot}66$; (2) $t = \frac{6}{{\cdot}366} = 16{\cdot}393$; (3) $t = \frac{20}{{\cdot}366} = 54{\cdot}645$.]

8. A mass of gas occupies 844 cubic inches at 15° C.; what space will it occupy at 0° C., under the same pressure? *Ans.* 800·8 cubic inches.

$$[V_o = \frac{V_t}{1+kt} = \frac{844}{1\cdot0549} = 800\cdot8.]$$

9. The coefficient of expansion of air being ·00366 for 1° C., find its coefficient of expansion for 1° F. *Ans.* ·00203.

10. A hundred cubic inches of alcohol, at 0° C., become 102·5 cubic inches at 25° C.; find the mean coefficient of expansion of alcohol between 0° C. and 25° C. *Ans.* ·001.

$$[V_t = V_o\,(1+kt) = V_o + V_o kt \therefore k = \frac{V_t - V_o}{V_o\,t} = \frac{2\cdot5}{100\times25} = \cdot001.]$$

11. A mass of gas occupies 864 cubic centimetres at 15° C.; what will be its volume at 40° C., the pressure remaining the same. *Ans.* 938·9 cubic inches.

[If V_o, V_t, V_m, represent the volume at 0°, t°, m°, respectively, and k the coefficient of expansion, then $V_t = V_o\,(1+kt)$; but $= V_o = \frac{V_m}{1+km} \therefore V_t = \frac{V_m}{1+km} \times (1+kt)$. Hence, in the present case, $V_t = \frac{864 \times (1 + \cdot00366 \times 40)}{1 + \cdot00366 \times 15} = 938\cdot9$.]

12. The volume of a certain mass of air, measured at 10° C., and at a pressure of 29 inches of mercury, is 100 cubic feet; what will be its volume at 0° C. and a pressure of 30 inches of mercury? *Ans.* 93·25 cubic feet.

[Boyle's law gives us the formula for change of volume due to change of pressure, when the temperature is constant. If V_P, V_H represent the volumes under the pressures P and H respectively, this law may be expressed $V_P = \frac{V_H\,H}{P}$. Again, the formula $V_o = \frac{V_t}{1 + kt}$ gives us the change of volume due to change of temperature, when the pressure is constant. By combining these formulas, as the student will be able to reason out for himself, we get a formula for the change of volume, when both temperature and pressure are changed. Let $V_{t\,H}$ represent the volume at any temperature t° C. and any pressure H, $V_{o\,P}$ the volume at 0° C. and any other pressure P; the formula will then be $V_{oP} = \frac{V_{tH}\;H}{P\,(1+kt)}$. Applying this formula to the present case, $V_{oP} = \frac{100 \times 29}{30 \times 1\cdot0366} = 93\cdot25$.]

13. In calculating the lifting power of the Paris Captive Balloon, it was assumed (p. 43) that the weight of air is 1·3 kilogrammes, and of hydrogen ·09 kilogrammes, per cubic metre. Now this is true only when the temperature is 0° C. and the pressure 760 millimetres.

Supposing then that the actual temperature, at the time of the ascent, is 20° C. and the pressure 750 millimetres, find what correction must be introduced into our former calculation. *Ans.* The lifting power will be reduced by 2,378·5 kilos.

[The volume of the balloon is $\frac{4}{3}\pi r^3$, which gives 24,429 cubic metres, r being 18 metres. This is evidently the volume of the air displaced, and also of the hydrogen that fills the balloon, both existing at a temperature of 20° C. and a pressure of 750 millimetres. To get the actual weight of the air displaced, and of the hydrogen, we must find what that volume would be reduced to at 0° C. and 760 millimetres pressure. For this purpose we use the formula of the last exercise, $V_{0\,P} = \frac{V_{t\,H}\,H}{P(1 + kt)} = \frac{24,429 \times 750}{760 \times 1\cdot0732} = \frac{18,321,750}{815\cdot632} = 22,463\cdot25$.

Hence the reduced volume is 22,463·25 cubic metres;
The weight of air displaced, 22,463·25 × 1·3 = 29,202·225 kilos.;
The weight of Hydrogen, 22,463·25 × ·09 = 2,021·6925 kilos.;
The resultant upward pressure, 27,180·5325 kilos.;

Whereas in former calculations we had

The weight of air displaced, 24,429 × 1·3 = 31,757·7. kilos.;
The weight of the Hydrogen, 24,429 × ·09 = 2198·61 kilos.;
The resultant upward pressure, 29,559·09.

Therefore the resultant upward pressure must be reduced from 29,559 to 27,180·5 kilos.; that is, by the amount of 2,378·5 kilos.]

MONTHLY EXAMINATION PAPER, No. 5.

HEAT, SECTIONS V., VI., VII.

A. Questions.

1. Give the temperature of fusion of half a dozen of the more important metals.

2. A block of ice, at a temperature of −10° C., is placed in a tin vessel over a spirit lamp; explain fully the successive changes of temperature and of condition which the ice undergoes before it is all converted into vapour.

3. Describe some experiments which illustrate the great force with which water expands, in freezing; and show the effects of this force in shaping the outline of mountain peaks and ridges.

4. Explain the physical process by which a snowball is made; and point out the reason why it is impossible to make a snowball when the snow is several degrees below freezing point.

5. How is the melting point of a solid affected by change of pressure? Explain your answer fully, both as regards the fact, and as regards the theory by which the fact is accounted for. State also the bearing of this fact on the opinion that the rocks within the crust of the earth exist in a state of igneous fusion.

6. Explain what is meant by ebullition or boiling; and describe an experiment by which it may be shown that the temperature at which a liquid boils depends on the pressure to which it is subjected.

7. Describe the successive changes that take place in the condition of steam, as it issues from the funnel of a locomotive; and explain the physical nature and cause of each change.

8. Describe the phenomenon known as the spheroidal state of liquids; and explain, as far as you can, the physical cause of it.

9. What is meant precisely by the humidity or moisture of the atmosphere, and in what does it consist? Is the moisture of the atmosphere always the same when the same quantity of vapour is contained in a given quantity of air?

10. Explain the physical process by which fires are effective in keeping a room dry on a damp day. Would an equally good effect be produced by lighting all the burners of a large gasalier?

11. Describe Regnault's Hygrometer, and explain the principle of its action, showing how, by its means, the dew point may be approximately ascertained.

12. After a clear day, about sunset, a fog frequently appears near the banks of a river; give a reason for this phenomenon, and explain why the fog does not usually extend more than a few feet above the ground.

13. On a warm day in summer I have seen a perfectly dry glass vessel filled with ice, and set on a dinner table in a crowded room; in half an hour the table cloth all round it was saturated with water. Explain the cause of this phenomenon.

B. Exercises.

1. The air is saturated with water vapour, the temperature being 0° C.; find the weight of vapour present in 10 cubic feet. *Ans.* 20·264 grains.

[In this and the following exercises the density of water vapour may be taken as $\frac{5}{8}$ of the density of dry air at the same pressure and temperature; and the weight of a cubic inch of dry air, at 0° C. and 760 millimetres pressure, may be taken as ·31 of a grain, which gives for the weight of a cubic foot, $\cdot 31 \times 1728 = 535\cdot 68$ grains.

The vapour, in the exercise given above, being in a state of saturation, is at its maximum tension for 0° C., which we find in the tables to be 4·6 millimetres. The weight of 10 cubic feet of vapour at this pressure and 0° C., is $\frac{5}{8}$ of the weight of 10 cubic feet of dry air at the same pressure and temperature. To find this, we must first ascertain what would be the volume of such a mass of air at a pressure of 760 millimetres. Using the formula $V_P = \frac{V_H H}{P}$, we get $V_P = \frac{10 \times 4\cdot 6}{760} = \cdot 0605263$ of a cubic foot; and this multiplied by 535·68 gives 32·4224 grains, for the weight of the air, which multiplied again by $\frac{5}{8}$, gives 20·264 grains for the weight of the vapour.]

2. The air is saturated with water vapour, and the temperature is 10° C.; find the weight of vapour present in 3 cubic feet. *Ans.* 11·685 grains.

[Here the tension of the vapour is equal to the maximum tension at 10° C., which, from the tables, is 9·165 millimetres. To find the weight of 3 cubic feet of dry air, at the same pressure and temperature, we may use the formula $V_{0P} = \frac{V_{tH} H}{P(1 + kt)}$, which will give us $\frac{3 \times 9\cdot 165}{760\,(1\cdot 0366)} \times 535\cdot 68 = 18\cdot 696$ grains; and this, multiplied by $\frac{5}{8}$, will give 11·685 for the weight of the vapour.]

3. The temperature of the air is 18° C., the dew point is 15° C.; find the weight of water vapour in 5 cubic feet. *Ans.* 26·24 grains.

[The tension of the vapour present, at any time, in the atmosphere, is equal to the maximum tension that corresponds to the temperature of the dew point. In the case before us the dew point is 15° C., and the maximum tension of water vapour, at 15° C., we find in the tables to be 12·699 millimetres. Consequently this is the actual tension of the vapour with which we have to deal, and its temperature is 18° C. We have now to find the weight of 5 cubic feet of dry air, at the same temperature and pressure, and to multiply it by $\frac{5}{8}$ in order to get the answer required. Taking the formula $V_{0P} = \frac{V_{tH} H}{P(1 + kt)}$, we get, for the weight of the air, $\frac{5 \times 12\cdot 699}{760\,(1\cdot 06588)} \times 535\cdot 68 = 41\cdot 985$ grains; and, for the weight of the vapour, $41\cdot 985 \times \frac{5}{8} = 26\cdot 24$.]

4. The air is saturated with the vapour of water; the temperature is 15° C., the pressure 750 millimetres. What is the tension of the air as distinguished from the vapour with which it is charged? *Ans.* 737·301 millimetres.

[The tension of a mixture of gas and vapour is equal to the sum of the tensions of each considered separately. In the present case the sum of the tensions of the air and the vapour balances the atmospheric pressure, and is therefore 750 millimetres; the tension of the vapour alone, since it exists in a state of saturation, is equal to the maximum tension of water vapour at 15° C.; this we find in the tables to be 12·699 millimetres; therefore the tension of the dry constituents taken alone is $750 - 12{\cdot}699 = 737{\cdot}301$ mm.]

5. **The temperature of the air is 15° C., the dew point 10° C., the pressure of the atmosphere is 745·25 millimetres; find the tension of the dry constituents of the air.** *Ans.* **736·085 millimetres.**

[The tension of the vapour present is equal to the maximum tension of water vapour at the temperature of the dew point, that is, at 10° C.; this, as we learn from the tables, is 9·165 millimetres; and therefore the tension of the dry constituents of the air is $745{\cdot}25 - 9{\cdot}165 = 736{\cdot}085$ millimetres.]

6. **Through a bent tube containing pumice stone, saturated with sulphuric acid, a cubic metre of air at 15° C. is passed, and the tube is found to weigh 3·95 grammes more than it did before. Find the hygrometric state of the air. In this and the following exercises the weight of a cubic metre of dry air, at 0° C., and 760 millimetres pressure, may be taken as equal to 1·3 kilogrammes.** *Ans.* **·307.**

[The hygrometric state of the air is the ratio of the quantity of vapour present, in a given volume, to the quantity necessary for saturation, at the same temperature. In the case before us, the sulphuric acid may be assumed to absorb all the vapour present in the cubic metre of air; and this quantity is given to us by the increased weight of the tube, namely 3·95 grammes. The problem then is, to find the ratio of this quantity to the quantity that would be necessary for the saturation of a cubic metre at 15° C. Now the tension of water vapour, existing in a state of saturation, at 15° C., is 12·699 millimetres, as we learn from the tables; and its weight will be $\frac{5}{8}$ of the weight of dry air at the same temperature and pressure. Using the formula $V_0 = \frac{V_{tH}H}{P(1+kt)}$, we get the weight, in kilogrammes, of a cubic metre of dry air, at 15° C., and 12·699 millimetres pressure, to be $\frac{1 \times 12{\cdot}699}{760 \times 1{\cdot}0549} \times 1{\cdot}3 = {\cdot}02059$; and this, multiplied by $\frac{5}{8}$, gives us ·012868 kilogrammes, or 12·868 grammes, for the weight of vapour in a cubic metre of saturated air at 15° C. Hence the hygrometric state of the air submitted to examination is $\frac{3{\cdot}95}{12{\cdot}868} = {\cdot}307$.]

7. **In an experiment similar to the last the weight of water vapour in a cubic metre of air at 10 C. is found to be 4·25 grammes; find the hygrometric condition of the air.** *Ans.* **·45.**

[The maximum tension of water vapour at 10° C. is 9·165 millimetres; and therefore, using the same formula as in the last exercise, we get the weight of a cubic metre of saturated vapour at 10° C., equal to $\frac{1 \times 9{\cdot}165}{760 \times 1{\cdot}036} \times 1{\cdot}3 \times \frac{5}{8} =$ ·00945 kilogrammes, or 9·45 grammes. Hence the hygrometric state of the air examined is $\frac{4{\cdot}25}{9{\cdot}45} = {\cdot}45$.]

8. In another similar experiment the quantity of aqueous vapour present in a cubic metre of air is 4 grammes, and the temperature 0° C.; find the hygrometric state of the air. *Ans.* ·813.

[The maximum tension of water vapour at 0° C. is 4·6 millimetres; and therefore the formula $\frac{V_{tH}\,H}{P\,(1 + kt)}$ becomes $\frac{4{\cdot}6}{760}$, and the weight of vapour required for saturation will be $\frac{4{\cdot}6}{760} \times 1{\cdot}3 \times \frac{5}{8} = {\cdot}004{,}918$ kilogrammes, or 4·918 grammes, which gives, for the hygrometric state of the air examined, $\frac{4}{4{\cdot}918} = {\cdot}813$.]

9. A mass of air extending over an area of 60,000 square metres, and rising to a height of 300 metres, has a temperature of 20° C., and the dew point is 15° C. Find how much vapour will be condensed into water if the temperature fall to 10° C. *Ans.* 57,600 kilogrammes.

[The tension of the vapour present in the air is equal to the maximum tension of water vapour at the dew point, which we learn from the tables to be 12·699 millimetres; and its temperature is 20° C. Therefore the weight of a cubic metre of it is equal to $\frac{5}{8}$ of the weight of a cubic metre of dry air, at 20° C., and 12·699 millimetres pressure. This will be, according to the formula already employed, $\frac{1 \times 12{\cdot}699}{760 \times 1{\cdot}0732} \times 1{\cdot}3 = {\cdot}02024$ of a kilogramme, or 20·24 grammes; and multiplying by $\frac{5}{8}$, we get 12·65 grammes for the weight of vapour present in a cubic metre of the air under examination. Now when the tempreature falls to 10° C., the weight of vapour necessary for the saturation of a cubic metre will be $\frac{1 \times 9{\cdot}165}{760\,(1{\cdot}0366)} \times 1{\cdot}3 \times \frac{5}{8} = {\cdot}00945$ of a kilogramme, or 9·45 grammes. Therefore the quantity of vapour that will be condensed into water, in each cubic metre, will be $12{\cdot}65 - 9{\cdot}45 = 3{\cdot}2$ grammes. This quantity, multiplied by the number of cubic metres in the mass, namely 18,000,000, will give 57,600 kilogrammes.]

10. The general temperature of the air being 10° C., what is the least humidity of the atmosphere sufficient for the deposit of dew on the grass, it being assumed that air close to the grass is 7° C. below the general temperature? *Ans.* 62.

[The hygrometric condition must be such that the air would be just saturated at 3° C. If E represent the hygrometric condition of the air, f the actual tension of the water vapour present in it, and F the tension of the water vapour that would be required for its saturation at the same temperature, then $E = \frac{f}{F}$ But, from the conditions of the question, f is equal to the maximum tension of water vapour at 3° C., which we find in the tables to be 5·687; and F is evidently the maximum tension at 10° C., which is 9·165; $\therefore\ E = \frac{5{\cdot}687}{9{\cdot}165} = {\cdot}62$.]

MONTHLY EXAMINATION PAPER, No. 6.

HEAT, SECTIONS VIII., IX.

A. Questions.

1. Give a definition of Latent Heat, illustrating your definition with examples; and state the latent heat of water and of steam.

2. Explain, as far as you can, what becomes of the heat that disappears when solids are melted and liquids are vaporized; and point out the bearing of this question on the doctrine of the Conservation of Energy.

3. Describe some method by which the latent heat of a liquid, for instance, water, may be determined.

4. Explain the theory of freezing mixtures, and give some examples of them.

5. A supersaturated solution of chloride of calcium has been carefully prepared in a Florence flask; a thermometer standing in the solution marks 10° C.; I take a minute particle of solid matter and drop it into the solution; in a moment the chloride of calcium is crystallized, and the thermometer goes up to 17° C. Explain this phenomenon.

6. Describe an experiment with the air pump in which a solid block of ice may be produced by the evaporation of ether; and explain the principle which the experiment illustrates.

7. Describe and explain Carré's machine for freezing water by its own evaporation.

8. Describe the means by which carbonic acid may be reduced to the liquid and the solid state; and explain, in each case, the physical causes that are in operation during the process.

9. Explain what is meant by Specific Heat, and illustrate your answer by examples. Give the specific heat of half a dozen familiar substances.

10. Define the meaning of a Unit of heat; and explain the different kinds of unit in common use.

[A unit of heat is the quantity of heat required to raise a unit mass of distilled water through a unit of temperature. As there are different units of mass, and different units of temperature, in use amongst scientific men, so there are different units of heat. If we take the pound weight for the unit of mass, and the degree Fahrenheit for the unit of temperature, then the unit of heat will be the quantity of heat necessary to raise a pound weight of water through one degree of the Fahrenheit scale. This may be called the Pound-Fahrenheit, or P.F. unit of heat. Again, if we take the pound weight as the unit of mass, and the degree Centigrade as the unit of temperature, then the unit of heat will be the quantity necessary to raise a pound weight of water through one degree of the Centigrade scale. This unit may be called the Pound-Centigrade, or P.C. unit of heat: it is evidently greater than the former in the same ratio in which the degree Centigrade is greater than the degree Fahrenheit, that is, in the ratio of 9 : 5. Once more, if we take the kilogramme as the unit of mass, and the degree Centigrade as the unit of temperature, then the unit of heat will be the quantity necessary to raise a kilogramme of water through one degree of the Centigrade scale. This unit may be called the Kilogramme-Centigrade, or K.C. unit of heat. It is greater than the P.C. unit in the same ratio in which the mass of a kilogramme is greater than the mass of a pound, that is, in the ratio of 2·2 : 1; and the student may easily satisfy himself that it is greater than the P.F. unit in the ratio of 9 × 2·2 : 5, or 3·96 : 1. These three units of heat are all in common use among scientific men, and the student should make it a point thoroughly to master their meaning and their mutual relations.]

11. Describe some method by which the specific heat of a solid body may be accurately determined.

12. Two precisely similar vessels are filled, one with water at 10° C., the other with mercury at 10° C., and placed over two precisely similar lamps. State what amount of heat each liquid must receive before it boils, and which will boil first. (London University).

[The water must be raised through 90° C.; therefore it must receive 90 pound-centigrade units of heat for every pound it weighs. The mercury must be raised through 340° C. since its boiling point is 350° C.; therefore, for every pound weight, it must receive $340 \times \cdot 033 = 11\cdot 22$ pound-centigrade units of heat, its specific heat being ·033. But the mercury weighs 13·6 times as much as the water; therefore, if the water weigh x lbs., the mercury must weigh $13\cdot 6 \times x$. Hence the total heat which the water must receive, before it boils, will be expressed, in pound-centigrade units, by $90x$; whereas the mercury must receive, before it boils, $11\cdot 22 \times 13\cdot 6 \times x = 152\cdot 6 \times x$ pound-centigrade units. The water will, therefore, boil first.]

13. I make three mixtures; one of ice at 0° C. and salt at 0° C. in equal quantities, another of ice at 0° C. and water at 0° C. in equal quantities, and a third of water at 0° C. and salt at 0° C. in equal quantities. Say, for each case, if there will be any change of temperature, and what change; also which of the mixtures will be the coldest.

B. Exercises.

1. How much heat will it take to melt a cubic metre of ice at 0° C., the specific gravity of ice being ·9? *Ans.* 71,325 K.C. units.

[As there are three different units of heat in common use, it will be found convenient to have a name and a symbol for each. I propose then to call them, in these exercises, the Pound-Fahrenheit or P.F. unit, the Pound-Centigrade or P.C. unit, and the Kilogramme-Centigrade or K.C. unit. The particular unit to be employed, in each case, will depend, in great measure, on the way in which the question is proposed. In the exercise before us, we have first to find the weight of a cubic metre of ice. The volume, in cubic decimetres, multiplied by the specific gravity, will give the weight in kilogrammes; that is, 1,000 × ·9 = 900. Then, since the Latent Heat of water is 79·25, in K.C. units, the heat required will be 79·25 × 900 = 71, 325.]

2. A kilogramme of water, at 50° C., is converted into ice at — 10° C.; how much heat is given out in the process? *Ans.* 134·25 K.C. units.

[The water gives out 50 K.C. units of heat in falling from 50° C. to 0° C.; 79·25 units in passing from water at 0° C. to ice at 0° C.; and the ice, in passing from 0° C to — 10° C. gives out 10 + ·5 = 5 units, the specific heat of ice being ·5. Therefore the heat given out during the whole process will be 50 + 79·25 + 5 = 134·25 K.C. units.]

3. How much heat is required to convert 10 kilogrammes of ice, at — 10° C., into water 20° C.? *Ans.* 1042·5 K.C. units.

[The heat required will be: (1) to raise the ice from — 10° C. to 0° C., 10 × ·5 × 10 = 50 K.C. units; (2) to melt 10 kilogrammes of ice at 0° C., 10 × 79·25 = 792·5 K.C. units; (3) to raise 10 kilogrammes of water from 0° C. to 20° C., 10 × 20 = 200 K.C. units; making in all 50 + 792·5 + 200 = 1042·5 K.C. units.]

4. How much heat is required to convert one pound of ice, at —10° C., into steam at 100° C.? *Ans.* 720·25 P.C. units.

[To raise a pound of ice from — 10° C. to 0° C. will take 10 × ·5 = 5 P.C units; to convert a pound of ice at 0° C. into water at 0° C. will take 79·25 P.C. units; to raise a pound of water from 0° C. to 100° C. will take 100 P.C. units; and lastly, to convert a pound of water at 100° C. into steam at 100° C. will take 536 P.C. units; making in all, 5 + 79·25 + 100 + 536 = 720·25 P.C. units.]

5. Five lbs. of mercury at 120° C. are mixed with one lb. of water at 14° C.; what will be the temperature of the mixture? *Ans.* 29° C.

[Let x represent the required temperature; $x - 14$ will represent, in P.C. units, the heat received by the water. This will be equal to the heat lost by the mercury. Now the mercury falls through $120-x$ degrees centigrade, and consequently parts with $(120-x)\ {\cdot}033 \times 5$ P.C. units of heat. Therefore $x - 14 = (120 - x) \times {\cdot}033 \times 5$; from which find $x = 29°$.]

6. Twelve pounds of mercury at 15° C. are mixed with six pounds

of iron filings at 50° C.; find the temperature of the mixture. The specific heat of iron may be taken at ·114. *Ans.* 37·1° C.

[Let x be the required temperature. If w, s, t, and w', s', t', represent the weight, the specific heat, and the temperature of the mercury and iron respectively, then the heat gained by the mercury will be $w \times s \times (x - t)$; and the heat lost by the iron will be $w' \times s' \times (t' - x)$. Now, from the conditions of the problem, these two quantities are equal; therefore $w \times s \times (x - t) = w' \times s' \times (t' - x)$, or substituting the values, $12 \times {\cdot}033 \times (x - 15) = 6 \times {\cdot}114 \times (50 - x)$, from which find $x = 37{\cdot}1$.]

7. A ball of iron, weighing 2 kilogrammes, at 100° C., is put into a cavity cut in a block of ice, at 0° C.; a cover of ice, also at 0° C., is laid over the cavity and the iron is allowed to cool. How much ice will be melted by the iron in cooling? *Ans.* 287·7 grammes.

[Let x be the weight, in kilogrammes, of ice melted. The heat imparted to the ice will be $79{\cdot}25 \times x$ in K.C. units. The heat lost by the iron will be $2 \times 100 \times {\cdot}114 = 22{\cdot}8$ K.C. units. These two quantities are evidently equal to each other, from the conditions of the problem; that is, $79{\cdot}25 \times x = 22{\cdot}8$ and $x = \frac{22{\cdot}8}{79{\cdot}25} = {\cdot}2877$ kilogrammes or 287·7 grammes.]

8. A kilogramme of ice, at — 10° C., is put into a kilogramme of linseed oil, at ·250° C.; what will be the temperature of the mixture? The specific heat of linseed oil may be taken at ·5, and of ice also at ·5. *Ans.* $27{\cdot}1\dot{6}$.

[If we take x to represent the required temperature, the linseed oil will lose $1 \times {\cdot}5 \times (250 - x)$ K.C. units of heat. The ice receives $10 \times {\cdot}5$ units in rising to 0° C., 79·25 units in passing from the solid to the liquid state, and x units in rising from 0° C. to x° C. $\therefore {\cdot}5 \times (250 - x) = 10 \times {\cdot}5 + 79{\cdot}25 + x$, and $125 - {\cdot}5x = 5 + 79{\cdot}25 + x$; which gives $x = 27{\cdot}1\dot{6}$.]

9. In what proportion must water, at 30° C., be mixed with linseed oil, specific heat ·5, at 50° C., in order to have 20 kilogrammes of the mixture at 40° C.? *Ans.* 6⅔ kilogrammes of water, and 13⅓ of oil.

[If x represent the weight of the water, in kilogrammes, $20 - x$ will be the weight of the oil. The heat received by the water will then be $10x$ K.C. units; and the heat given out by the oil will be $(20 - x) \times {\cdot}5 \times 10$ units. From the conditions of the problem these two quantities are equal, that is, $(20 - x) \times {\cdot}5 \times 10 = 10x$, and $x = 6\frac{2}{3}$, $20 - x = 13\frac{1}{3}$.]

10. Half a pound of steam, at 100° C., is let into 12 lbs. of water, at 20° C.; what will be the temperature of the mixture? *Ans.* 44·64° C.

[If x represent the required temperature, then $\frac{536}{2} + \frac{100 - x}{2}$ will represent, in P.C. units, the heat given up by half a pound of steam (1) in passing from the condition of steam at 100° C. into water at 100° C., and (2) in falling from 100° C.

to $x°$ C. On the other hand, 12 $(x-20)$ will represent, in P.C. units, the heat received by 12 lbs. of water, in rising from 20° C. to $x°$ C. From the conditions of the problem these two are equal, that is, $\frac{536}{2}+\frac{100-x}{2}=12(x-20)$, which gives $x=44\cdot64$.]

11. If I want to melt a cubic metre of ice at 0° C., by means of steam at 100° C., how much water must I convert into steam for the purpose ? *Ans.* 112·114 kilogrammes.

[A cubic metre of ice weighs $100 \times \cdot9 = 900$ kilogrammes. To melt 900 kilogrammes of ice I must expend $79.25 \times 900 = 71{,}325$ K.C. units of heat. In doing this work the steam will pass from steam at 100° C. to water at 0° C. Hence, if x be the weight, in kilogrammes, of steam required, it is plain that $536x+100x=71{,}321$, as each kilogramme of steam will give up 536 K.C. units of heat in passing into water at 100° C., and 100 units in passing from water at 100° C. to water at 0° C. Hence $x=\frac{71{,}321}{636}=112\cdot114$.]

12. Find the ratio of the Pound-Fahrenheit, the Pound-Centigrade, and the Kilogramme-Centigrade units to one another. *Ans.* 5 : 9 : 19·8 or 1 : 1·8 : 3·96.

13. How much heat will be required to convert 20 lbs. of water at 60° C. into steam at 100° C. ? *Ans.* 11,520 P.C. units.

MONTHLY EXAMINATION PAPER No. 7.

HEAT, SECTIONS X., XI.

A. Questions.

1. Describe an experiment by which the feeble conducting power of water for heat may be demonstrated.

2. Explain how it is that woollen clothes are eminently fitted to keep the human body warm, and yet a woollen covering will keep ice from melting on a hot day in summer.

3. Why is it that the stone pavement, in a Turkish bath, feels hotter than the rugs and carpets; whereas, in an ordinary house, it feels colder ?

4. Explain what is meant by the specific conducting power of a substance; and illustrate your answer by the case of iron, of which the specific conducting power is given as ·0011.

[By the specific conducting power of a substance is meant the facility with which it allows heat to flow through it. It is measured by the number of units of heat that will flow, in a unit of time, through a piece of the substance whose length is the unit of length, whose cross section is the unit of area, and whose opposite ends are kept at a constant difference of a unit of temperature. As there are various systems of units in use, among scientific men, the specific conducting power of the same substance may be differently expressed, according to the units employed. If the Pound-Centigrade unit be taken as the unit of heat, the inch as the unit of length, the degree Centigrade as the unit of temperature, and the second as the unit of time, then the specific conducting power of iron will be expressed by the figure ·0011; which means that, if we take a piece of iron one inch long, and one square inch in sectional area, and if we keep its opposite ends at a constant difference of temperature equal to one degree Centigrade, the quantity of heat that will pass every second from one end to the other will be ·0011 of a Pound-Centigrade unit.]

5. Two adjoining rooms are separated by a door: the temperature of the air in one room is 22° C., in the other, 10° C. Opening the door wide I hold a candle in the doorway; first near the top, then near the bottom, then about the middle. Describe the appearance the flame of the candle will present, in each case, and the cause of that appearance.

6. Describe the origin and course of the Trade Winds, and explain their physical cause.

7. State what is meant by Radiant Heat; explain the physical process in which it consists; and give a short list of good radiators and bad radiators.

8. Describe Melloni's apparatus for investigating the laws of Radiant Heat.

9. Explain Prevost's theory of exchanges, and illustrate your explanation with examples.

10. Why is it that, after a clear night, the blades of grass are often covered with dew, while the gravel walk beside the grass is dry?

11. Two similar vessels of polished block tin, one of which is covered with a tight fitting jacket of flannel, the other being left uncovered, are filled with boiling water, and suspended from hooks in the same room. In which of them will the water have cooled most, at the end of 20 minutes, and why?

12. Explain, as fully as you can, why the air is generally much colder on the top of a mountain than it is in the valley below.

13. Define the meaning of *luminous* and *obscure* heat rays; explain, as far as you can, the intrinsic difference between them; and describe some experiment by which the power of *obscure* heat rays may be strikingly exhibited.

14. Compare the action of a plate of glass and a plate of rock-salt on obscure heat rays; and show how your answer may be demonstrated by Melloni's apparatus.

B. Exercises.

1. It has been determined by experiment that the conducting power of silver, compared to that of iron, is as 100 to 11·9; it has further been determined by experiment that the specific conducting power of iron is ·0011; find the specific conducting power of silver. *Ans.* ·00924.

$$[x : {\cdot}0011 :: 100 : 11{\cdot}9 \;\therefore\; x = \frac{{\cdot}11}{11{\cdot}9} = {\cdot}00924.]$$

2. The conducting power of copper, compared to that of iron, is as 77·6 : 11·9; find the specific conducting power of copper. *Ans.* ·007173.

$$[x : {\cdot}0011 :: 77{\cdot}6 : 11{\cdot}9 \;\therefore\; x = \frac{77{\cdot}6 \times {\cdot}0011}{11{\cdot}9} = {\cdot}007173.]$$

3. Conceive an iron boiler, half an inch thick, in which the temperature of the inner surface is maintained permanently at 140° C., and that of the outer surface at 110° C.; and find how much heat will pass, every hour, through each square foot of surface. *Ans.* 34214·4 P.C. units.

[As the plates of the boiler are only half an inch thick, the quantity of heat that passes through each square inch of surface in a second will be ·0011 × 2 × 30 = ·066 P.C. units. This continued for an hour will be ·066 × 60 × 60 = 237·6 units for each square inch of surface, or 237·6 × 144 = 34214·4 units for each square foot.]

4. A copper boiler, of which the plates are $\frac{3}{8}$ of an inch thick, exposes an area of 10 square feet; the inner surface is maintained at a constant temperature of 100° C., and the outer at 99·5° C.; find how much heat will pass through the plates of the boiler in an hour. *Ans.* 49579·8 P.C. units.

[The specific conducting power of copper was found above to be ·007173. Hence bearing in mind that, in the case before us, the difference of temperature between the opposite surfaces is only ·5° C., and the thickness of the plates $\frac{3}{8}$ of an inch, the heat lost will be ${\cdot}007173 \times 5 \times \frac{8}{3} \times 60 \times 60 \times 144 \times 10 = 49579{\cdot}8$.]

5. Take the case of an iron boiler whose plates are $\frac{1}{3}$ of an inch thick, and whose opposite surfaces are kept at 120° C. and 100° C., respectively; how much heat will be lost every hour for each square foot of surface? *Ans.* 34214·4 P.C. units.

$$[{\cdot}0011 \times 3 \times 20 \times 60 \times 60 \times 144 = 34214{\cdot}4.]$$

MONTHLY EXAMINATION PAPER, No. 8.

HEAT, SECTIONS XII., XIII., XIV.

A. Questions.

1. Describe the fly wheel of a steam engine; explain the purpose for which it is used, and the way in which that purpose is effected.

2. Explain, in a general way, the chief improvements introduced by Watt in the construction of the steam engine.

3. Explain the difference between condensing and non-condensing engines, and show the advantages of the former.

4. Give an example of the production of heat by the expenditure of mechanical energy; and explain, as far as you can, the physical process by which it is produced.

5. Give an example of the production of heat by the expenditure of chemical energy, and explain the physical process, as far as you can.

6. Give an example of the production of heat by the expenditure of electrical energy; and compare the physical process in this case with the physical process in the two preceding cases.

7. What is to be thought of the saying that Oxygen is not itself *combustible*, but that it is a *supporter* of combustion; whereas Hydrogen is not a supporter of combustion, but is itself *combustible* ?

8. Explain the process of combustion as it takes place in an ordinary candle.

9. Describe Bunsen's burner, and explain the principle of its action.

10. Show that all the water power in the world may be traced to the energy of the Sun's heat.

11. State briefly the two prevailing theories on the nature of heat; and sum up the arguments in favour of the dynamical theory.

12. Describe and account for the experiment by which Count Rumford succeeded in boiling water by the heat generated in the boring of a cannon.

13. Explain what is meant by the mechanical equivalent of heat; state what it is, in foot pounds, and also in kilogrammetres, and describe some of the experiments by which it was determined.

14. State the law of the Conservation of Energy, and give some striking illustrations of it from the phenomena of heat.

B. Exercises.

1. In a non-condensing steam engine the area of the piston is 2 square feet, the length of stroke is 3·5 feet, the pressure is 60 lbs. per square inch, and the steam is not cut off till the stroke is complete. Find the work done on the piston, in each stroke. *Ans.* 45,360 foot lbs.

[Pressure $60 - 15 = 45$ lbs. per square inch, acting on 288 square inches, through 3·5 feet, gives $45 \times 288 \times 3{\cdot}5 = 45{,}360$.]

2. Under the same conditions, find what the work done, per stroke, would be in a condensing engine, the "vacuum" in the condenser having a pressure of 2 lbs. per square inch. *Ans.* 58,464 foot lbs.

[$(60 - 2) \times 288 \times 3{\cdot}5 = 58{,}464$.]

3. Suppose, in the last case, that the length of the stroke were doubled, and the steam cut off at the end of half the stroke: the quantity of steam used in each stroke would be the same as before; find what would be the work done on the piston, if the mean pressure of the steam, during the second half of the stroke, be 40 lbs. per square inch. *Ans.* 96,768 foot lbs.

[The work done in the first half of the stroke will be the same as the total work done before; in the second half, it will be $(40 - 2) \times 288 \times 3{\cdot}5 = 38{,}304$. Total work per stroke, $58{,}464 + 38{,}304 = 96{,}768$.]

4. Find how much heat energy must be expended to lift one ton weight to a height of 100 feet. *Ans.* 290·15 P.F. units, or 161·15 P.C. units.

[The work to be done is $2{,}240 \times 100 = 224{,}000$ foot lbs., and the heat required will be $\frac{224{,}00}{772} = 290{\cdot}15$ P.F. units, or $\frac{224{,}000}{1390} = 161{\cdot}15$ P.C. units.]

5. A block of stone, weighing half a ton, falls through a vertical height of 50 feet; find the heat generated when its motion is arrested. *Ans.* 72·5 P.F. units, or 40·3 P.C. units.

[The kinetic energy acquired by the stone in falling is equal to the energy necessary to lift it through 50 feet, or $50 \times 1120 = 56{,}000$ foot lbs. This energy is converted into heat when the motion of the stone is arrested, and yields $\frac{56000}{772} = 72{\cdot}5$ P.F. units, or $\frac{56000}{1390} = 40{\cdot}3$ P.C. units.]

6. A mass of rock, weighing 1,500 lbs., falls from the top of a precipice, 500 feet high, into the valley below. Find the amount of heat that is generated by the fall. *Ans.* 971·5 P.F. units, or 539·6 P.C. units.

[The potential energy, converted into kinetic energy by the fall, is 1,500 × 500 = 750,000 foot lbs.; and this, converted into heat, will give $\frac{750{,}000}{772} = 971{\cdot}5$ P.F. units, or $\frac{750{,}000}{1390} = 539{\cdot}6$ P.C. units.]

7. A cannon ball, weighing 68 lbs., strikes a target with a velocity of 1,400 feet a second; what will be the amount of heat generated in the collision? *Ans.* 2697·5 P.F. units, or 1498·2 P.C. units.

[The kinetic energy of a moving mass is $\frac{Wv^2}{2g}$ which in this case will be $\frac{68 \times (1400)^2}{64} = 2{,}082{,}500$ foot lbs. This energy, when wholly converted into heat, will yield $\frac{2{,}082{,}500}{772} = 2697{\cdot}5$ P.F. units, or $\frac{2{,}082{,}500}{1390} = 1498{\cdot}2$ P.C. units.]

8. If one-half of the heat generated in the last exercise were developed in the cannon ball itself, through how many degrees Centigrade would its temperature be raised? *Ans.* 96·63° C.

[If t represent the increase of temperature in degrees Centigrade, m the mass in pounds, and c the specific heat of the cannon ball, then $mtc = \frac{1498{\cdot}2}{2}$, and $t = \frac{1498{\cdot}2}{2mc} = \frac{1498{\cdot}2}{2 \times 68 \times {\cdot}114} = 96{\cdot}63$.]

9. A drop of rain falls to the ground from the height of a mile. Supposing all the heat generated by its fall to be concentrated in the drop itself, find how much its temperature would be raised. *Ans.* 3·8° C.

[The kinetic energy acquired by a pound of rain, in falling through a mile, would be 5280 foot lbs., which converted into heat would be $\frac{5280}{1390} = 3{\cdot}8$ P.C. units. This amount of heat would raise a pound weight of rain through 3·8° C.; and the heat generated by the fall of a drop would evidently have the same effect on the temperature of the drop.]

10. How much heat is produced per hour in a manufactory in which 1·2 horse-power of the driving force is expended in overcoming friction? *Ans.* 3077·7 P.F. units, or 1709·3 P.C. units.

[The kinetic energy converted into heat each hour is $33{,}000 \times 1{\cdot}2 \times 60 = 2{,}376{,}000$ foot lbs., which will yield $\frac{2{,}376{,}000}{772} = 3077{\cdot}7$ P.F. units, or $\frac{2{,}376{,}000}{1390} = 1709{\cdot}3$ P.C. units.]

11. What is the ratio between the heat generated when a bullet, weighing 50 grammes, strikes a target, with a velocity of 500 metres,

and the heat generated when a cannon ball, weighing 40 kilogrammes, strikes a target, with a velocity of 400 metres? *Ans.* 1 : 512.

[The heat generated in the two cases will be in the same ratio as the kinetic energy, that is, as $\frac{Wv^2}{2g} : \frac{W'v'^2}{2g}$ or as $Wv^2 : W'v'^2$ or as $\cdot 05 \times 250{,}000 : 40 \times 160{,}000$ or as 1 : 512.]

12. In the experiments at Meppen, in the summer of 1879, an 18-ton Krupp gun discharged a steel shot, weighing 352 lbs., with a velocity of 1,910 feet per second. Find how much heat would be developed if the shot were suddenly stopped when travelling with that velocity; and how much the temperature of the shot would be raised if all this heat were concentrated in the shot itself. The specific heat of the shot may be taken at ·12. *Ans.* (1) 14·435 P.C. units; (2) $341\cdot\dot{6}^\circ$ C.

[The kinetic energy of the shot is given by the formula $\frac{Wv^2}{2g}$ or $\frac{352}{64} \times (1910)^2 = 20{,}064{,}550$ foot lbs. This energy converted into heat will give $\frac{20{,}064{,}550}{1390} = 14{,}435$ P. C. units. This heat, distributed over 352 lbs., will be $\frac{14{,}435}{352} = 41$ units to each lb.; and will therefore raise the temperature of the mass through $\frac{41}{12} = 341\cdot\dot{6}^\circ$ C.]

13. How much heat is expended by a man, weighing 12 stone, in lifting his body to the top of a mountain 12,000 feet high? *Ans.* 1450 P.C. units.

[The work done is $12 \times 14 \times 12000 = 2{,}016{,}000$ foot lbs. To do this work the heat required will be $\frac{2{,}016{,}000}{1390} = 1450$ P.C. units.]

N.B.—It should be observed that the lifting of the body is only a small portion of the work done in actually climbing a mountain.

GENERAL EXAMINATION PAPER ON SOUND.

A. Questions.

1. Describe the experiments by which the velocity of sound in air has been determined; and state what that velocity is at any two different temperatures you please to select.

2. Explain, as far as you can, how the velocity of sound, in air, is affected by the heat developed by condensation, in one half, and

the cold developed by rarefaction, in the other half, of the sound wave itself.

3. Explain and prove the formula $L = \frac{V}{n}$, L being the length of a wave of sound produced by a given note, n the number of vibrations of the note, per second, and V the velocity of sound.

4. Arrange the following substances in order, according to the velocity of sound in each: water, iron, oak, hydrogen, air; and explain what it is that causes the difference of velocity.

5. Explain as fully as you can why it is that, in the case of an echo, the sound appears to come not from the reflecting surface, but from a considerable distance behind it.

6. Explain what is meant by Intensity, Pitch, and Quality, in musical tones, and state in what each consists.

7. Write down the notes of the musical scale, and give the rate of vibration of each, in reference to the first or fundamental note.

8. Explain how you would proceed to determine the rate of vibration of a beetle's wings, by means of the siren.

9. Explain, as far as you can, the physical difference between speaking and singing.

10. State how the length of the sound waves, produced by a note of a given pitch, is affected by a change of temperature in the air, and give a reason for what you say.

11. Describe the apparatus known as Trevelyan's Rocker, and explain the physical cause of the musical note it produces.

12. Explain the terms Major Tone, Minor Tone, Semitone; state precisely in what each of them consists, and how many of each are contained in the gamut.

[These terms are used to designate the several kinds of interval between adjacent notes of the diatonic scale. Considered as a relation between two sensations, the interval between two musical notes may be felt but cannot be described. Considered as an objective phenomenon, it consists in the rate of vibration of the one as compared to the other. Thus the interval between the notes Do Re is the ratio 9 : 8; and the same interval is found between the notes Fa Sol, and also between the notes La Si. This interval is called a Major Tone. Between the notes Re Mi, the ratio is 10 : 9, and the same ratio is found between the notes Sol La. This interval is called a Minor Tone. The Semitone consists in the ratio 16 : 15, and is found between the notes Mi Fa, and between the notes Si Do. Thus there are three Major Tones, two Minor Tones, and two Semitones in the complete scale.]

13. State what you know regarding Newton's calculation, founded

on theoretical considerations, of the velocity of sound; and explain, as far as you can, the reason of the notable discrepancy between the result of his calculation and the observed fact.

14. Explain what is meant by Temperament in the tuning of a piano, and why it is necessary.

15. State the relation that exists between the length of an organ pipe, open and closed, and the length of the sound wave of its fundamental note; and trace back this relation to its fundamental cause.

16. Describe the sonometer, and explain how it may be employed to prove experimentally that the rate of vibration of a stretched string is inversely proportional to its length, $n \propto \frac{1}{L}$.

17. Describe Melde's apparatus for illustrating the laws of vibrating strings; and explain the experiments by which, with this apparatus, it may be proved that the rate of vibration is proportional to the square root of the tension of the string, $n \propto \sqrt{T}$.

18. If a tuning fork is put in vibration, and held close to the ear, there is a definite position of the tuning fork in which no sound is heard, and another position in which the sound heard has a maximum intensity; show, with the aid of a diagram, what these two positions are, and explain the physical cause of the phenomenon.

19. Describe Kœnig's apparatus for the analysis of musical sounds by means of the resonators of Helmholtz and the system of manometric flames.

20. Describe generally the structure of the human ear; and trace the course of sound waves from their entrance into the outer chamber until they strike upon the nerve of hearing.

B. Exercises.

1. The report of a pistol is returned by an echo, after a lapse of 3 seconds; what is the distance of the reflecting surface from the pistol, the temperature being 10° C.? *Ans.* 1665 feet.

[The velocity of sound in air, at a temperature of 10° C., is approximately 1110 feet a second; therefore, in 3 seconds, it will travel $1110 \times 3 = 3330$ feet, and the reflecting surface is evidently half that distance, or 1665 feet.]

2. A bullet is discharged from a rifle, with a velocity of 1200 feet a second, and at the end of 2·5 seconds, the person who has fired the rifle hears the bullet strike the target, the temperature, at the

time, being 0° C.; find the distance of the target, supposing the bullet to have retained its initial velocity undiminished. *Ans.* 1428 feet.

[Let x represent the distance in feet; then $\frac{x}{1200}$ is the time, in seconds, that the bullet takes to reach the target, and $\frac{x}{1090}$ is the time, in seconds, that the sound takes to travel back. Then, from the conditions of the problem, $\frac{x}{1200} + \frac{x}{1090} = 2{\cdot}5$, and $x = 1428$.]

3. If a bell is struck at the surface of a lake, and its sound reflected from the bottom is heard after 1·2 seconds, what is the depth of the lake? *Ans.* 2821·2.

[Sound travels in fresh water at the rate of 4,708 feet a second; therefore it travels $4{,}702 \times 1{\cdot}2 = 56424$ feet in 1·2 seconds, and the bottom of the lake will be at a distance of $\frac{5642{\cdot}4}{2} = 2821{\cdot}2$ feet.]

4. The bell in the clock tower at Westminster is 300 feet above the ground, and an observer stands at a point 400 feet from the base of the tower; find what time it will take for the sound of the bell to reach his ear, on a day when the temperature is 15° C. *Ans.* ·4464 of a second.

[Make a diagram and show that the distance of the observer from the bell, which will be the base of a right angle triangle, whose small sides are 300 and 400 feet respectively, is 500 feet; hence the time required will be $\frac{500}{1120} = {\cdot}4464$ of a second.]

5. If the Vartry water pipe were exposed for a length of half a mile, and were struck forcibly with a hammer at one end of this distance, a person listening at the other end would hear the blow twice; what would be the interval between his two perceptions of the sound, the temperature being 20° C.? *Ans.* 2·179 seconds.

[The velocity of sound in iron, at the temperature of 20° C., is 16,822 feet a second, and, in air, it is 1130 feet a second. Hence the sound will reach the observer first through the iron conductor, and then, after an interval, through the air. The distance being $880 \times 3 = 2640$ feet, the sound will travel through the iron conductor in $\frac{2640}{16{,}822} = {\cdot}157$ of a second, and through the air in $\frac{2640}{1130} = 2{\cdot}336$ seconds. The interval between the two perceptions will therefore be $2{\cdot}336 - {\cdot}157 = 2{\cdot}179$.]

6. Assuming the fundamental Do of the diatonic scale to be produced by 256 vibrations in a second, write down the number of vibrations per second for each successive note of the scale.

[Do 256, Re $256 \times \frac{9}{8} = 288$, Mi $256 \times \frac{5}{4} = 320$, Fa $256 \times \frac{4}{3} = 341\frac{1}{3}$, Sol $256 \times \frac{3}{2} = 384$, La $256 \times \frac{5}{3} = 426\frac{2}{3}$, Si $256 \times \frac{15}{8} = 480$, Do $256 \times 2 = 512$.]

7. By what interval does a major exceed a minor tone? *Ans.* $\frac{81}{80}$.

[A major tone is $\frac{9}{8}$, a minor tone $\frac{10}{9}$; therefore the interval by which the former exceeds the latter is the ratio $\frac{9}{8} : \frac{10}{9}$, or 81 : 80. This interval is called a comma.]

8. A wire, stretched by a weight 13 lbs., emits a musical note when put in vibration; what must be the stretching weight, applied to the same wire, to make it produce the major third above that note? *Ans.* 20·2135 lbs.

[The rate of vibration is proportional to the square root of the tension, or stretching weight. Hence if n and n' represent the rate of vibration of the two notes respectively, T and T′ the corresponding tensions of the wire, $n : n' :: \sqrt{T} : \sqrt{T'}$. But $n : n' :: 4 : 5$, $\therefore \sqrt{T} : \sqrt{T'} :: 4 : 5$, or $T : T' :: 16 : 25$, and $T' = T\frac{25}{16} = 13 \times \frac{25}{16} = 20{\cdot}2135$.]

9. The diameters of two wires, of the same length and the same material, are ·0015 and ·0038 millimetres respectively, and their stretching weights 400 and 1600 grammes respectively; find the ratio of their rates of vibration. *Ans.* $1{\cdot}2\dot{6} : 1$.

[The rate of vibration of a stretched wire is directly proportional to the square root of the tension, and inversely proportional to the radius of the wire. Taking the same symbols as before, with R and R′ to represent the radii of the two wires respectively, we have, $n : n' :: \frac{\sqrt{T}}{R} : \frac{\sqrt{T'}}{R'} :: \frac{20}{{\cdot}00075} : \frac{40}{{\cdot}0019} :: 1{\cdot}2\dot{6} : 1$.]

10. A brass and a silver wire, of the same diameter, are stretched by weights of 2 and 25 lbs. respectively, and produce the same note; find the ratio of their lengths, the density of brass being 8·39, and of silver 10·47. *Ans.* The length of the silver wire is to the length of the brass wire as 3·1615 : 1.

[Taking n, T, L, D, and n', T′, L′, D′, to represent the rate of vibration, tension, length, and density, of the silver and brass wires respectively, we have, from the laws for traverse vibrations of strings, $n : n' :: \frac{\sqrt{T}}{L\sqrt{D}} : \frac{\sqrt{T'}}{L'\sqrt{D'}}$; but, from the conditions of the problem, $n = n'$, $\therefore \frac{\sqrt{T}}{L\sqrt{D}} = \frac{\sqrt{T'}}{L'\sqrt{D'}}$, and $\frac{L\sqrt{D}}{\sqrt{T}} = \frac{L'\sqrt{D'}}{\sqrt{T'}}$. Hence $L = L' \times \frac{\sqrt{D'}}{\sqrt{T'}} \times \frac{\sqrt{T}}{\sqrt{D}} = L' \times \frac{\sqrt{8{\cdot}39}}{\sqrt{2}} \times \frac{\sqrt{25}}{\sqrt{10{\cdot}47}}, = L' \times 3{\cdot}1615$.]

11. A tuning fork vibrates 545 times a second: what is the length of a closed organ pipe which, at 0° C., will resound most perfectly to the note of the fork, the length of the organ pipe being assumed to be great in proportion to its diameter? *Ans.* 6 inches.

[The organ pipe that will resound most perfectly is one-fourth the length of the sound wave produced by the note, that is, $\frac{L}{4}$; but $L = \frac{V}{n}$, or, in the present case, $\frac{1090}{545} = 2$ feet; and the organ pipe is, therefore, 6 inches.]

GENERAL EXAMINATION PAPER ON HEAT.

A. Questions.

1. State the reasons why mercury is usually chosen for thermometers in preference to other liquids. What are the advantages and disadvantages of alcohol as compared with mercury?

2. What is meant by the displacement of Zero in a thermometer? What is the cause of this displacement? and how may it be provided against?

3. Explain the construction and principle of some form of deep sea thermometer.

4. Describe the phenomenon of Regelation; and discuss the theory by which it is to be accounted for.

5. Explain the process by which a glacier is formed.

6. A block of ice, at 0° C., is subjected to an increase of pressure equal to 5 atmospheres; state if it will undergo any change (1) of condition, or (2) of temperature, and give a reason for each part of your answer.

7. A flask of Vartry water at 15° C., with a thermometer in it, resting on the bottom, is placed over a spirit lamp, and, in a few minutes, the water is boiling; describe the process that has gone on in the water, during that time, and the indications of the thermometer.

8. I find that the thermometer, mentioned in the last question, registers 102° C., when the water is boiling, and not 100° C.; can you assign any cause for this?

9. A flask half full of water is placed over a spirit lamp, until the water boils; the lamp being then removed, and the flask corked, the

water ceases to boil and begins to cool. If I now turn the flask upside down, and squeeze *cold* water on it from a sponge, the water begins to boil again. Explain the cause of this phenomenon.

10. I have a flask of water and a flask of ether, at exactly the same temperature; why is it that, if I pour a few drops of each on my hand, in succession, the ether feels much colder than the water?

11. Explain how it is that a bottle of wine, in summer, if wrapped in a wet cloth, and placed in a draught, is rapidly cooled.

12. Why is water, in summer, if kept in a vessel of porous earthenware, cooler than if kept in a vessel of glazed earthenware?

13. Explain what is meant by a saturated vapour, and a non-saturated vapour, and give examples of each. In which sense may oxygen and hydrogen be regarded as vapours, and why?

14. Describe the method by which Faraday first produced liquid carbonic acid.

15. State generally the phenomena of Latent Heat; and point out the bearing of these phenomena on the two theories regarding the nature of Heat.

16. I take 2 lbs. of pounded ice at 0° C., and one pound of common salt at 10° C.; when I mix them together I find that the temperature of the mixture is -18 C. Explain, as fully as you can, the physical cause of the cold here produced.

17. Give some examples of the importance, in the economy of nature, of the great Latent Heat and the great Specific Heat of water, and of the vapour of water.

18. Describe some method by which the Latent Heat of Steam may be determined.

19. Describe an experiment in which water *rises* in temperature, in the very act of freezing; and explain the physical cause of the phenomenon.

20. Explain what is meant by the Specific Heat of a gas under *constant pressure*, and its Specific Heat at a *constant volume;* state the relation that exists between the two; and show how the excess of one above the other might be predicted from the doctrine of the Conservation of Energy.

21. Explain precisely what is meant by the dew point, and how it is connected with the humidity or moisture of the atmosphere.

22. Describe the wet and dry bulb Hygrometer; and explain how, by its means, the dew point may be determined approximately.

23. Explain what is meant by the maximum tension of a vapour at

a given temperature; and show that the actual tension of the water vapour present, at any time, in the atmosphere, is equal to the maximum tension of water vapour at the temperature of the dew point.

24. Explain what is meant by the hygrometric state of the atmosphere; and show that, if d and f represent respectively the density and tension of the water vapour actually present in the atmosphere, D and F the density and tension that would be necessary for saturation, at the same temperature, and E the hygrometric state, then

$$E = \frac{d}{D} = \frac{f}{F}.$$

25. Explain the construction and principle of Davy's Safety Lamp.

26. What is meant by Radiant Heat? Explain the physical process of radiation; and give a short list of good radiators and bad radiators.

27. How is it that the leaves of trees are often quite dry on a morning on which the grass, round about, is covered with dew?

28. What is hoar frost? Why is it formed generally on clear nights rather than on cloudy nights? And how is it that it is often found on grass and on the leaves of plants when the general temperature of the air is several degrees above freezing point?

29. Why is it that the linen sheets of a bed feel cold and the blankets warm though both are at exactly the same temperature?

30. Explain what is meant by saying that a steam engine "works expansively," and show by an example the advantages of this system.

31. Describe the construction and explain the action of the Governor in a steam engine.

32. Explain the term horse-power; and state its value in terms of foot pounds, and also in terms of kilogrammetres.

33. What is meant by *incandescence*, and how is it distinguished from *combustion?* Explain, as far as you can, the physical phenomenon in each case.

B. Exercises.

1. Convert the following readings of the Fahrenheit scale into the corresponding readings Centigrade: (1) 70° F., (2) 54° F., (3) −39° F. *Ans.* (1) $21{\cdot}\dot{1}^{\circ}$ C., (2) $12{\cdot}\dot{2}^{\circ}$ C., (3) −39·44° C.

2. Convert the following readings of the Centigrade scale into the corresponding readings Fahrenheit: (1) 21° C., (2) 11° C., (3) −9° C. *Ans.* (1) 69·8° F., (2) 51·8° F., (3) 15·8° F.

3. Convert the following readings of the Fahrenheit scale into readings of the Réaumur scale: (1) 45° F., (2) 30° F., (3) −7° F. *Ans.* (1) $5{\cdot}\dot{7}^\circ$ R., (2) 2 − $0{\cdot}\dot{8}^\circ$ R., (3) $-17{\cdot}\dot{3}^\circ$ R.

4. Convert the following readings of the Centigrade scale into readings of the Réaumur scale: (1) 16° C., (2) −15° C. *Ans.* (1) 12·8° R., (2) −12° R.

5. Find at what temperature the readings of the Fahrenheit and Réaumur scales exactly correspond. *Ans.* −25·6°

[Since $R = (F - 32)\frac{4}{9}$ it follows that, when R = F, $(F - 32)\frac{4}{9} = F$, or $\frac{4}{9}F - \frac{4}{9}32 = F \therefore 4F - 128 = 9F$, and $F = -\frac{128}{5} = -25{\cdot}6$.]

6. The length of a brass rod at 0° C. is 15 feet; find its length at 25° C., the coefficient of linear expansion of brass, for one degree Centigrade, being ·00001875. *Ans.* 15·00703125 feet.

[$l_t = l_0(1 + kt)$; that is, in our case, $l_t = 15 \times (1 + {\cdot}00001875 \times 25) = 15{\cdot}00703125$.]

7. The railway from Dublin to Cork is 165 miles in length; what is the difference between the aggregate length of the rails forming a single line of the railway, at the lowest winter temperature, say — 10° C., and at the highest summer temperature, say 50° C.? The coefficient of linear expansion of the metal may be taken at ·0000122. *Ans.* 211·2 yards.

[Taking k for the coefficient of linear expansion, and t for the extreme difference of temperature, the actual expansion, expressed in terms of the length at 0° C., would be, for one mile, kt or $\cdot0000122 \times 60 = \cdot000732$; and for a length of rail that would measure 165 miles at 0° C., it would be $\cdot000732 \times 165 = \cdot12$ nearly. Now the aggregate length of the rails in question is approximately 165 miles at 0° C., and therefore the total range of expansion for a change of temperature from − 10° C. to 50° C. will be ·12 miles or 211·2 yards.]

8. The diameter of an iron wheel is 6 feet at 0° C.; what will be the actual increase in length of its circumference if raised to 400° C. Coefficient of expansion ·0000122. *Ans.* ·092 feet.

9. What is the volume at 200° C. of a mass of lead which, at 0° C., has a mass of one cubic foot, the coefficient of expansion of lead being ·0000285? *Ans.* 1·0171 cubic feet.

10. If a wire, 120 inches long at 0° C., becomes 121 inches long at 270° C., what is its coefficient of cubical expansion? *Ans.* ·0000926.

[If l be the length at 0° C., k the coefficient of linear expansion, and t the increase in temperature, the total expansion is $lkt = 1$ inch, $\therefore\ k = \frac{1}{lt} = \frac{1}{120 \times 270} =$ ·000030867; and this, multiplied by 3, gives ·0000926, the coefficient of cubical expansion.]

11. The specific gravity of lead at 0° C. is 11·352, find its specific gravity at 65° C., taking the coefficient of linear expansion for lead at ·00002855. *Ans.* 11·289.

[Let $S_0\ S_t$, $V_0\ V_t$, represent the specific gravity and the volume at 0° C. and $t°$ C. respectively; then, $\frac{S_t}{S_0} = \frac{V_0}{V_t} = \frac{V_0}{V_0(1 + kt)} = \frac{1}{1 + kt}\ \therefore\ S_t = \frac{S_0}{1 + kt}$. In the present case, $k = {·}00002855 \times 3 = {·}00008565$, and this multiplied by 65 gives ·00556725 $\therefore\ S_t = \frac{11{·}352}{1{·}00556725} = 11{·}289$.]

12. A pendulum is composed of a platinum rod passing through a zinc sphere, which rests on a flattened surface at the end of the rod. What must be the diameter of the sphere as compared with the length of the rod in order that the centre of the sphere shall be at the same distance from the point of supension under all changes of temperature? The coefficient of linear expansion of platinum is ·0000088, and of zinc ·0000294. *Ans.* Equal to ·6 of the length of the rod, at 0° C.

[Let l be the length of the platinum rod at 0° C., and k its coefficient of expansion, r the radius of the sphere at 0° C., and k' its coefficient of expansion, and let t be any other temperature. Then the change of length of the platinum rod for any change of temperature, say to $t°$ C., will be lkt, and the change in length of the radius of the sphere, for the same change in temperature, will be $rk't$. It is evident that the former change will tend to move the centre of the sphere in one direction and the latter in the opposite direction. Therefore if $lkt = rk't$, the centre of the sphere will not be moved by any change of temperature; in other words, the conditions of the problem will be fulfilled when $lkt = rk't$, or when $r = l\frac{k}{k'} = l \times \frac{{·}0000088}{{·}0000294} = l \times {·}3$.]

13. A gas has a volume of 100 cubic centimetres at 10° C., and a pressure of 740 millimetres; find its volume at 0° C., and a pressure of 760 millimetres. *Ans.* 93·93 cubic centimetres.

$$[V_{0P} = \frac{V_{tH}\,H}{P\,(1 + kt)} = \frac{100 \times 740}{760\,(1{·}0366)} = 93{·}93.]$$

14. A litre of gas is measured off, at 20° C., and a pressure of 750 millimetres; find its volume at 0° C., and a pressure of 760 millimetres. *Ans.* ·9195 litre.

$$[V_{0P} = \frac{V_{tH}\,H}{P\,(1 + kt)} = \frac{1 \times 750}{760 \times (1{·}0732)} = {·}9195.]$$

15. A mass of gas at 40° C. has a volume of 200 cubic centimetres; what will be its volume, under the same pressure, at 10° C.? *Ans.* 180·84 cubic centimetres.

[If V_t V_m V_0 represent the volume at $t°$ C., $m°$ C., 0° C., $V_t = \frac{V_m}{1+km} \times (1+kt)$; and, in the present case, $V_t = \frac{200}{1\cdot1464} \times 1\cdot0366 = \frac{207\cdot32}{1\cdot1464} = 180\cdot84$.]

16. Fifty cubic centimetres of a gas are cooled down, under constant pressure, from 10° C. to —10 C.; what is the resulting volume? *Ans.* 46·47 cubic centimetres.

$$[V_t = \frac{V_m}{1+km} \times (1+kt) = \frac{50}{1\cdot0366} \times \cdot9634 = \frac{48\cdot17}{1\cdot0366} = 46\cdot47.]$$

17. The barometer stands at 780 millimetres, when the temperature is 24° C.; find the corresponding reading for the temperature of 0° C. *Ans.* 776·64 millimetres.

$$[h_0 = \frac{h_t}{1+kt} = \frac{780}{1+\cdot00018 \times 24} = \frac{780}{1\cdot00432} = 776\cdot64.]$$

18. The temperature of the air is 20° C., the dew point is 12° C.; what is the hygrometric state of the atmosphere? *Ans.* ·6.

[If E represent the hygrometric state of the atmosphere, q, d, f, the quantity, density, tension, respectively, of the vapour actually present in a given volume, Q, D, F, the quantity, density, tension of the vapour that would be required for the saturation of that volume at the existing temperature, then $E = \frac{q}{Q} = \frac{d}{D} = \frac{f}{F}$. The student, having convinced himself of the truth of these equations, must next realize the fact that f will be always equal to the maximum tension of water vapour at the temperature of the dew point, and that F will be the maximum tension at the existing temperature. The tables give him both; and thus, in the present case, he will find $E = \frac{10\cdot457}{17\cdot391} = \cdot6$.]

19. The temperature of the air is 18° C., the dew point 16° C.; what is the hygrometric state of the atmosphere? *Ans.* ·88.

20. The dew point is 4° C., the temperature 6° C.; what is the hygrometric state of the atmosphere? *Ans.* ·87.

21. Four lbs. of copper, at 130° C., are placed in 20 lbs. of water at 20° C., and the temperature of the water is found to be raised 2° C. Find the specific heat of copper from these data. *Ans.* ·0926.

[If S represent the specific heat of the copper, the heat given out by it, in cooling to 22° C., will be $4 \times 108 \times S$; and the heat gained by the water is 20×2. From the conditions of the problem these quantities are equal, that is,

$432\,S = 40$ and $S = \frac{40}{432} = \cdot0926$. This result is somewhat below the true specific heat of copper, because, in the experiment described, some heat is, of necessity, lost by radiation and conduction.]

22. A pound of platinum is placed in a furnace, and when it has acquired the temperature of the furnace, it is plunged in a vessel containing 10 lbs. of water at 10° C.; the temperature of the water rises to 14·3° C. Find the temperature of the furnace, the specific heat of platinum being taken at ·032. *Ans.* 1358·05° C.

[If x° C. be the temperature of the furnace, the platinum loses $(x - 14\cdot3) \times \cdot032$ P.C. units of heat; while the water gains $10 \times 4\cdot3$ P.C. units; therefore $(x - 14\cdot3) \times \cdot032 = 10 \times 4\cdot3$; from which find $x = 1358\cdot05$° C.]

23. One kilogramme of ice at 0° C., is mixed with 2 kilogrammes of water at 80° C.; find the temperature of the mixture. *Ans.* 26·916° C.

[If x represent the required temperature $79\cdot25 + x = 2\,(80 - x)$, from which $x = 26\cdot91\dot{6}$.]

24. Half a pound of oil, specific heat ·5, at 50° C., is mixed with one pound of water at 10° C.; what is the temperature of the mixture? *Ans.* 18° C.

$$[\cdot5 \times \cdot5 \times (50 - x) = x - 10 \therefore x = 18.]$$

25. A kilogramme of ice at 0° C. is put into 2 kilogrammes of water at 0° C.; how much steam at 100° C. will be required to melt the ice and raise the temperature of the mixture to 30° C.? *Ans.* 279 grammes.

[Let x be the weight of the steam in kilogrammes; the latent heat of steam being 536, the steam at 100° C., passing to water at 30° C., will give out $536 \times x + 70 \times x$ K.C. units of heat. This will be equal to what the ice and water receive, namely, $79\cdot25 + 3 \times 30$ or $169\cdot25$ K.C. units. Hence $536x + 70x = 169\cdot25$, and $x = \cdot279$ kilogrammes.]

26. A kilogramme of mercury at 0° C. is added to a kilogramme of water at 100° C.; find the temperature of the mixture. *Ans.* 96·8° C.

27. A litre of mercury at 0° C. is added to a litre of water at 100° C.; find the temperature of the mixture. *Ans.* 69° C.

28. A pound of steam at 100° C. is mixed with 10 lbs. of water at 20° C.; what is the temperature of the mixture? *Ans.* 76° C.

[If x represent the required temperature, $536 + (100 - x) = 10\,(x - 20)$, from which $x = 76$.]

29. The steam from 10 lbs. of boiling water is conducted into a vessel containing 120 lbs. of water at 0° C.; find the resulting temperature. *Ans.* 48·92° C.

30. How many pounds of steam at 100° C. will just melt 12 lbs. of ice at 0° C.? *Ans.* 1·495 lbs.

$[536x + 100x = 79{\cdot}25 \times 12 \therefore x = 1{\cdot}495.]$

31. A bar of copper, specific heat ·095, weighing 5 lbs., is taken from a furnace and plunged into 25 lbs. of water at 10° C., which it raises to 29° C.; find the temperature of the furnace. *Ans.* 1029° C.

[If x represent the temperature of the copper, which may be assumed to be the same as that of the furnace, $5 \times {\cdot}095 \times (x - 29) = 20 \times (29 - 10)$, which gives $x = 1029$.]

32. Six lbs. of mercury, at 50° C., are mixed with 8 lbs. of water at 100° C.; what is the temperature of the mixture? *Ans.* 98·79° C.

33. Two ounces of platinum, taken from a furnace, are plunged into 26 ounces of water at 0° C., and the temperature of the water rises to 3° C.; find the temperature of the furnace, the specific heat of platinum being ·035. *Ans.* 1117·3° C.

34. It is estimated that, on an average, 100,000,000 tons of water come down every hour, through a vertical height of 150 feet, at the Falls of Niagara. Find how many steam engines, each yielding 100 horse power, would be necessary to furnish the same amount of energy. *Ans.* About 170,000.

[The kinetic energy of the falling water is $100{,}000{,}000 \times 150 = 15{,}000{,}000{,}000$ foot tons per hour. Now one horse power gives, per hour, $33{,}000 \times 60 = 1{,}980{,}000$ foot lbs., or $\frac{1{,}980{,}000}{2240} = 884$ foot tons of energy. Therefore the horse power required will be $\frac{15{,}000{,}000{,}000}{884} = 17{,}000{,}000$ nearly; and the number of steam engines, each yielding 100 horse-power, will be 170,000.]

35. If each steam engine, in the above example, burned 3 lbs. of coal per horse power per hour, how much coal should be annually consumed, to yield an energy of steam power equal to the energy of the falling water at Niagara? *Ans.* About 200,000,000 tons.

[N.B.—This is about equal to the total amount of coal consumed annually in the whole world.]

36. A lead bullet falls to the ground through a height of 100 metres: find how much its temperature would be raised if all the heat generated by its fall were concentrated in the bullet itself. The specific heat of lead may be taken at ·0314. *Ans.* 7·516° C.

[The kinetic energy of a kilogramme of lead, falling through 100 metres, is 100 kilogrammetres, which, converted into heat, will give $\frac{100}{424} = \cdot236$ of a K.C. unit. This heat imparted to the kilogramme of lead will raise its temperature through $\frac{\cdot236}{\cdot0314} = 7\cdot516°$ C. It is evident that the heat generated by the fall of a bullet will produce the same effect on the temperature of the bullet.]

37. A block of stone, weighing one ton, falls from a height of 300 feet upon a glacier, of which the temperature is 0° C. If all the heat generated by the fall were imparted to the ice, how much of the ice would be melted? *Ans.* 6·1 lbs.

38. How much heat is generated when a bullet, weighing 50 grammes, strikes against a target, with a velocity of 50 metres a second? *Ans.* ·015 K.C. units.

[The kinetic energy of the bullet may be got in kilogrammetres by the formula $\frac{Wv^2}{2g}$, if we take W in kilogrammes, v in metres, and put $g = 9\cdot8$. This gives the kinetic energy $\frac{\cdot05 \times (50)^2}{19\cdot6} = 6\cdot38$ kilogrammetres, which, converted into heat, will be $\frac{6\cdot38}{424} = \cdot015$ K.C. units. This heat would raise 1 kilogramme of water through ·015° C., or 1 gramme through 15° C.]

39. A meteoric stone, of one pound weight, enters our atmosphere with a velocity of 20 miles a second, and loses one-tenth of its velocity by friction in passing through; how much heat is generated? *Ans.* 1253·5 P.C. units.

[The velocity destroyed is 2 miles or $2 \times 1760 \times 3 = 10{,}560$ feet a second. Hence the kinetic energy converted into heat, $\frac{Wv^2}{2g}$, is $\frac{1}{64} \times (10560)^2 = \frac{111{,}513{,}600}{64} = 1{,}742{,}400$ foot lbs. This energy will yield $\frac{1{,}742{,}400}{1390} = 1253\cdot5$ P.C. units of heat.]

40. If one-half the heat generated, in the above example, were developed in the meteoric stone itself, how much would its temperature be raised? The specific heat of the meteoric stone may be taken at ·12. *Ans.* 5223° C.

[$\frac{1253\cdot5}{2} = 626\cdot75$ P.C. units, which will raise the meteoric stone through $\frac{626\cdot75}{\cdot12} = 5223°$ C.]

APPENDIX I.

EXAMINATION PAPERS OF THE INTERMEDIATE EDUCATION BOARD FOR IRELAND, JUNE, 1879.

[Questions marked thus * are known as Extra Questions, and are intended to test more advanced knowledge than that which is required for a Simple Pass.]

THEORETICAL MECHANICS.

1. State and explain the proposition known as the parallelogram of forces.

Show that if three forces acting on a particle produce equilibrium—

α. They act in the same plane.

β. Each is proportional to the sine of the angle between the lines of action of the other two.

2. Enunciate the proposition known as the triangle of forces.

Two cords, A C length 44 inches, B C length 117 inches, are attached to two points, A and B, in the same horizontal line, and to a weight of 25 lbs. at C; the angle A C B is a right angle; find the tension in each cord.

3. Find the resultant of two concurrent parallel forces acting on a rigid body.

A bar A B, 5 feet 4 inches in length, rests horizontally with its ends on two vertical props A and B. A weight of 320 lbs. is suspended from the bar at a distance of 27 inches from A; find the pressure on each prop.

4. Explain the different systems of levers, and give examples of each kind.

A man carries a bundle weighing 6 lbs. at the end of a stick a yard long resting on his shoulder; how will the pressure on his shoulder be altered by moving the place where the stick rests from its middle point 3 inches nearer the bundle?

5. Explain the principle of the common windlass, and calculate the ratio of the power of the weight.

The two radii of a compound wheel and axle are 4 inches and 3 inches respectively, and the radius of the wheel is 30 inches; neglecting friction and the rigidity of the cord, find the "*mechanical advantage*" which this wheel and axle can afford.

6. Find the conditions of equilibrium of any number of forces acting on a particle in the same plane.

Forces of $3 \times \sqrt{2}$ lbs., 4 lbs., 5 lbs., 7 lbs., and 2 lbs. respectively, act on a particle in the same plane; the angle between the first and second is 45°, the angles between the second and third, the third and fourth, and fourth and fifth, are each 90°; show that the particle is in equilibrium.

7. Explain the principle of the composition of velocities.

Two straight railway lines cross at right angles; a train moving at the rate of 40 miles an hour is approaching the crossing on one line, and is half a mile distant from it, when a second train on the other line, moving at the rate of 30 miles an hour, is passing through it. How far apart will these trains be at the end of five minutes, and when will they be nearest to each other.

8. Find the space (s) described in time (t) by a particle moving from rest with a uniform acceleration (f).

A ball projected vertically upwards with a certain velocity ascends to a height of 144·9 feet; find that velocity, and the time which will elapse before the ball returns to the point of projection. [The resistance of the air is supposed to be neglected, and the acceleration of gravity to be 32·2 feet per second.]

9. State the Second Law of Motion, as laid down by Newton, and mention any facts which illustrate it.

Suppose the ball in the preceding question to weigh 4 ounces, and its initial velocity to have been generated by a constant force acting on it for two seconds, find the weight which this force could sustain.

* 10. Determine the position of the centre of gravity of a system of material particles lying in the same plane, the respective distances of the particles from two straight lines in the plane, which intersect at right angles, being given.

A B C is a right-angled triangle; weights of 15 lbs., 14 lbs., and 12 lbs. respectively are placed at A, B, and C; given A B = 80 inches, A C = 89 inches, and B C = 39 inches, find the distance from A of the common centre of gravity of the weights.

* 11. Find the acceleration of a heavy particle sliding down a smooth inclined plane of given inclination.

Show that the times of sliding from rest down all smooth cords of a vertical circle which pass through its highest point are the same.

Are the velocities acquired the same?

* 12. Two weights, P and Q, are connected by a fine inextensible string which passes over a smooth pulley. If $P > Q$, find the acceleration with which the weights will move.

To one end of a fine string, arranged as above, weights of 49 ounces and 4 ounces respectively are attached; to the other end a weight of 51 ounces. After the system has moved from rest for two seconds the weight of 4 ounces is detached; when will the motion be reversed?

NATURAL PHILOSOPHY.

Junior Course:

EXPERIMENTAL MECHANICS, HYDROSTATICS, PNEUMATICS.

1. What is meant by a vertical line at a point on the surface of the earth?

2. How would you determine the centre of gravity of a common poker?

3. Explain and give a sketch of any kind of pulley.

4. If a marble and a cork were dropped from a window at the same time, which would reach the ground first?

5. A cog wheel with 13 teeth, and going round twice in a minute, works a cog wheel with 91 teeth; how fast will the latter go?

6. If the specific gravity of iron be 7·2, what is the weight of a cubic foot of iron? (N.B.—A cubic foot of water weighs 62·5 lbs.)

7. How much less would the cubic foot of iron weigh if it were immersed in water?

8. Explain how a fountain acts.

9. Describe how a waterwheel is worked.

10. Draw a section of a common pump.

11. Explain why a balloon goes up in the air.

12. If three cubic feet of air, at a pressure of 20 lbs. on the square inch, be compressed into two cubic feet, what will the pressure become?

13. Explain the action of a diving bell.

14. Describe the construction of a barometer.

* 15. Why does a train move slowly at starting?

* 16. What is the property of the common pendulum on which its utility depends?

* 17. Explain the action of a siphon.

* 18. How is specific gravity determined?

Middle Course:

SOUND AND HEAT.

1. How is the velocity of sound measured?

2. On what does the pitch of a note depend?

3. Explain the production of an echo.

4. What is the use of a sounding board attached to a musical instrument?

5. Describe an experiment to show that bodies are expanded by heat.

6. Why has a thermometer a bulb and a thin stem?

7. Explain the draught up a chimney.

8. Why does water boil more easily on the top of a mountain than it does at the level of the sea?

9. Draw a section of the cylinder and slide valve of a steam engine.

10. Explain the process of distillation.

11. Why do water pipes burst in frost?

12. What is meant by specific heat?

13. If a pound of ice at 0° C., and a pound of water at 80° C., be mixed together, what will be the temperature of the mixture?

14. Why does a drop of ether on your hand feel very cold?

* 15. Describe how a thermometer is filled.

* 16. Explain what is meant by the mechanical equivalent of heat.

* 17. What are the laws of the tension of vapours?

* 18. Explain why the fundamental note of a closed organ pipe is an octave below that of an open one.

SENIOR COURSE:

LIGHT, ELECTRICITY, AND MAGNETISM.

1. State the laws of reflection of light.
2. Show how to find the principal focus of a mirror.
3. What is meant by the refractive index of a substance?
4. Describe the phenomenon of the total reflection of light.
5. What is meant by the solar spectrum?
6. Draw a section showing the construction of the eye.
7. What is the difference between a magnet and a magnetic substance?
8. What is meant by the variation (declination) of the compass?
9. How would you magnetise a needle?
10. Describe an experiment to show that there are two kinds of electricity.
11. Give a list of conductors and non-conductors.
12. Explain the ordinary frictional electric machine.
13. Describe any form of voltaic battery.
14. Explain the action of a galvanometer.

* 15. Explain the formation of images by lenses.
* 16. How has the velocity of light been determined?
* 17. Explain the action of a Ruhmkorff's coil.
* 18. Describe any form of electric telegraph.

APPENDIX II.

ON A NEW FORM OF VOLTAIC BATTERY SUITABLE FOR INTERMEDIATE SCHOOLS AND COLLEGES.

[Abstract of a Paper read before the Royal Dublin Society, on Monday, April 15th, 1878.]

Every one who is engaged in physical or chemical researches must, I think, have often felt the want of a constant Voltaic Battery, always ready for immediate use, and capable of giving a strong electric current. The battery which I have the honour to bring under your notice this evening is intended to meet this want. I have called it a new *form* of battery, because, in fact, there is nothing new about it except the form. The elements employed are zinc and carbon, and the exciting liquid is a solution of bichromate of potash in dilute sulphuric acid, prepared according to the directions of Professor Bunsen, of Heidelberg. It is, therefore, essentially Bunsen's Bichromate of Potash battery, but it seems to me far more effective, for the purpose I have mentioned, than any form of that battery at present in use. I will first briefly describe its construction, and then I will give you an estimate of its efficiency, and show it to you in action.

The cell that I have employed is one of very peculiar character. It was invented and patented, some years ago, by Messrs. Fitzgerald and Molloy, of London, and is called by them the Multicarbon Cell. The carbon element is composed of five plates of carbon, each being about seven inches in length by two in breadth. These carbon plates are imbedded in lead, and by this means firmly attached to a circular plate of iron, thus forming a sort of hollow prism, which fits down in a large earthenware jar, on the upper edge of which the iron rim is supported. This arrangement secures two important points: first, a large carbon surface, and secondly, a very perfect contact between the carbon plates and the positive binding screw of the cell. The zinc element is simply a rectangular plate of amalgamated zinc, about the same size as each one of the carbon plates.

This cell is intended by its inventors to be used in the form of a double fluid battery. Hence they furnish, for an inner cell, a cylindrical porous vessel, in which the zinc element is placed. They recommend dilute sulphuric acid as the exciting agent in the porous cell, and, for the oxydizing agent, in the outer cell, a solution of chromate of calcium in sulphuric acid. About two years ago, one of the inventors was good enough to send me a specimen of this cell complete: I had heard it favourably spoken of, and wished to make experiments with it. I tried it first according to his directions, using in the outer cell a solution of chromate of calcium, of which he had sent me a supply, and, in the porous cell, dilute sulphuric acid. But I soon found that, when employed in this way, it was not suitable for the object I had in view. I wanted a permanent battery, that might stand for months in the Lecture Hall, ready at any moment for immediate use, and capable of giving a current strong enough for all ordinary purposes. Now the battery I have just described failed in two respects: first, it was not permanent; the chemical action of the cell went on when it was not in use, and it would work itself out in a few days; secondly, it was not capable of furnishing a strong electric current, owing to the great internal resistance caused by the porous cell.

Nevertheless it seemed to me that this cell, though unsuited to my purpose, when used as a double fluid battery, offered very special advantages for a single fluid battery. These advantages are, 1° the unusually large surface of the carbon element, 2° the exceedingly convenient and effective connection between the carbon plates and the positive binding screw of the cell, and 3° the large size of the cell itself, which diminishes the internal resistance, and affords room for an abundant supply of the oxydizing agent. I therefore determined to discard the porous vessel, and to try the cell simply as a Bichromate of Potash battery. The result more than fulfilled my expectations. I found that the cell, employed in this way, had an electro-motive force somewhat greater than a cell of Grove's battery, while the internal resistance was exceedingly small: and what surprised me not a little was, that the cell worked with far greater constancy than I had ever before seen exhibited by this kind of battery. After a few days of experiment, I came to the conclusion that a battery of six of these cells would answer all the ordinary purposes of the Laboratory and the Lecture Hall; I gave my plan to Mr. Yeates, of Dublin, and he has produced, with his usual skill, the very neat and workmanlike apparatus that you see before you.

You will observe that the zinc plates are all attached to one moveable frame, supported by strings which pass over pulleys; and that the whole is counterbalanced by weights concealed within these two hollow pillars, in the middle of the stand. By means of a handle, the zinc plates can be plunged into the liquid, and withdrawn at pleasure. The cells rest on a platform, which is supported on castors, so that the battery, which I must say is very heavy when charged, can be easily moved along the floor of the Laboratory or Lecture Hall. Here, then, we have a battery which, with moderate use, will remain in good working order for many months, which is able to yield a powerful electric current, which gives out no unpleasant fumes, and which is ever at hand, ready for immediate use.

In estimating the efficiency of a battery three things are chiefly to be considered, its electro-motive force, its internal resistance, and its constancy. It is well known that, in its ordinary form, the Bichromate of Potash battery has a high electro-motive force combined with a small internal resistance; and this battery before you shares in these advantages. Its electro-motive force is about two

Volts, and its internal resistance I estimate, after many experiments, at somewhat less than one tenth of an Ohm. But in respect of constancy it excels any other form of the Bichromate battery with which I am acquainted. It loses its power more slowly when in action, and recovers it more rapidly when at rest. These special qualities I ascribe mainly to the large surface of the carbon plates, and the great quantity of the oxydizing agent employed.

As might naturally be expected, its constancy varies very much according to the amount of resistance introduced into the external circuit. When the resistance of the external circuit is great, the battery will lose but a small fraction of its power in the space of three quarters of an hour: when, on the other hand, the external resistance is small, its power will fall off, to the same extent, in a few minutes. But I think I shall best give you a practical idea of the efficiency of this battery by saying that, for all the ordinary purposes of the Lecture Hall, it is fully equal to a Grove battery of six quart cells. With this battery I have obtained a spark of five inches and a quarter from a large induction coil; whereas, with a Grove battery of six quart cells, very carefully prepared with fresh nitric acid, I could obtain only four inches and a quarter from the same coil. Again, in decomposing water, this battery produced 100 cubic centimetres of the mixed gases in 60 seconds; while the Grove battery took 76 seconds to do the same work under the same conditions. In like manner this battery was the more effective of the two in charging an electro magnet, and in heating a spiral of platinum wire; and it was, at least, equally effective in producing a brilliant star of light between the fine points of two carbon pencils. Experiments of this kind, which may be said fairly to represent the power of a battery under varying conditions of external resistance, seldom need to be prolonged more than three or four minutes at a time; and, during that period, the battery before you loses but little of its power. A few moments rest restores it to its former strength, and it is then ready for a new experiment.

So much for the efficiency of this battery as compared with that of Grove: in point of convenience there is really no comparison. A Grove battery must be put together, and charged with acid, on each several occasion on which it is required for use; whereas this battery is always ready for use, and is put into action by a motion of the hand. A Grove battery is exhausted after a few hours' work; whereas this battery, with moderate work, will last for months together without any considerable loss of power. One or two other points of minor importance are not, I think, altogether undeserving of attention. This battery affords special facilities for connecting the cells together in "quantity" arrangement or in "tension" arrangement, as the saying is, according as best suits the purpose in hand. Again, we are in a position to alter, at will, the internal resistance of the battery, changing, by a movement of the hand, the depth to which the zinc plates are immersed in the bichromate solution. Thus we are always able, with this battery, to get a current exactly suited for the work we want to do, and the materials of the battery are only consumed to the extent necessary for accomplishing that work.

In conclusion, I wish to remind you again that this battery has no claim to be regarded as a new invention. It is simply a modification of Bunsen's Bichromate of Potash battery: and the chief value of the modification consists in the employment of a cell which has been already several years in existence, though it has not been hitherto used in this combination.

[For the successful action of this battery it is important, in making the bichromate of potash solution, to observe exactly the proportions laid down by Professor Bunsen. These proportions are as follows: To make 10 litres of the solution, take ·92 kilogrammes of commercial bichromate of potash in the state of fine powder; ·935 litres of concentrated sulphuric acid; and ·9 litres of water.

The powdered bichromate of potash may be placed first in a large jar or trough; the sulphuric acid is then slowly added, the mass being constantly stirred, until it is brought into the condition of a thick paste. The water should now be poured in, through a spout half an inch wide, and the stirring meanwhile continued. The whole mass, during this process, will gradually pass into a liquid state, and assume a bright ruby colour: when cool it will be fit for use.]

APPENDIX III.

BOOKS RECOMMENDED FOR THE USE OF COLLEGES.

TEXT BOOKS FOR ADVANCED BOYS.

Ganot, Elêments de Physique: or the English translation by Atkinson.

Deschanels, Traité Elémentaire de Physique: or the English translation by Everett.

Drion et Ferney, Traité de Physique Elémentaire.

TEXT BOOKS FOR JUNIOR BOYS.

Ganot, Popular Natural Philosophy.
Balfour-Stewart, Lessons in Elementary Physics.
Weinhold, Experimental Physics.

GENERAL BOOKS OF REFERENCE.

Jamin, Cours de Physique.
Pickering, Physical Manipulation.
Thomson and Tait, A Treatise on Natural Philosophy.
Secchi, L'Unité des Forces Physiques.
Helmholtz, Populäre Wissenschaftliche Vorträge.
Herschel, Familiar Lectures on Scientfic Subjects.
Airy, Popular Astronomy
Huxley, Physiography.
Grove, The Correlation of Physical Forces.
Balfour-Stewart, On the Conservation of Energy.
Tait, Recent Advances in Physical Science.

SPECIAL BOOKS OF REFERENCE.

(*a*) MECHANICS AND HYDROSTATICS.

Ball, Experimental Mechanics.
Lardner, Handbook of Natural Philosophy: Mechanics, Ed. 1877.
Clerk-Maxwell, Matter and Motion.
Bottomley, Dynamics, or Theoretical Mechanics.
Magnus, Elementary Mechanics.
Haughton, Manual of Mechanics.
Newth, Mechanics and Hydrostatics.
Besant, Elementary Hydrostatics.

(*b*) SOUND.

Tyndall, Sound, A Course of Eight Lectures.
Helmholtz, Die Lehre von den Tonempfindungen.
Herschel, Article on Sound in the Encyclopædia Metropolitana.
Airy, On Sound and Atmospheric Vibrations.
Lord Rayleigh, Theory of Sound.

(*c*) HEAT.

Tyndall, Heat, a Mode of Motion.
Balfour-Stewart, An Elementary Treatise on Heat.
Clerk-Maxwell, Theory of Heat.
Rankine, Manual of the Steam Engine.

(*d*) LIGHT.

Tyndall, Six Lectures on Light, delivered in America.
Lommel, The Nature of Light.
Lloyd, Wave Theory of Light.
Roscoe, Spectrum Analysis.
Schellen, Die Spectralanalyse.
Spottiswoode, Polarization of Light.
Secchi, Le Soleil.

(*e*) ELECTRICITY AND MAGNETISM.

Faraday, Experimental Researches.
Clerk-Maxwell, A Treatise on Electricity and Magnetism.
Tyndall, Diamagnetism.
Ferguson, Electricity.
Jenkin, Electricity and Magnetism.
Sabine, History and Progress of the Electric Telegraph.
Culley, Handbook of Practical Telegraphy.
Preece and Sivewright, Telegraphy.

(*f*) METEOROLOGY.

Scott, Weather Charts and Storm Warnings.
Scott, On the use of Meteorological Instruments.
Herschel, Meteorology.

BROWNE AND NOLAN, PRINTERS, DUBLIN.